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# Variable-Gain Cross-Coupling Controller for Contouring

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**Summary :**

The increasing trend for higher precision in manufacturing systems has brought an increasing interest in the design of servo controller. One of the most effective methodologies for contouring applications is the cross-coupling control (CCC). This paper introduces a new CCC method which utilizes variable gains that are adjusted in process according to the shape of the part. The variable-gain CCC enables a contour error reduction of 3:1 to 10:1, depending upon the starting point and the resolution of the system as well as the contour type. The paper presents analysis and simulation results.

**Keywords :** Control system, Controller.

## 1. Introduction

To achieve the high precision required for future machining trends, a more accurate servo controller system is needed. Conventional systems, using proportional (P) controller, cause contour errors [1,2,10]. These errors might be at the acceptable range (e.g., 0.05 mm = 0.002 inch) when machining at present feedrates (e.g., 2 mm/sec = 5 ipm). However, since the contour errors are proportional to the feedrate value, they will become enormously large in high-speed machining which requires high feedrates. We should emphasize that the **contour error**, rather than the positioning error is the prime concern, although the latter is usually given as the specification of CNC systems (typical value 0.01 mm).

In conventional multi-axis contouring systems, each axis is controlled by a separate control loop. Contour errors are caused by (1) the differences in loop parameters, (2) disturbance loads, and (3) the contour shape in nonlinear cuts. These errors are due to the drive system and the controller itself. Additional error sources are in the mechanical hardware : backlash, friction, etc. Reduction of contour errors by non-conventional techniques is being performed by two basic approaches: (1) Improving the dynamic response of each individual axis, and (2) cross-coupling control.

The first approach is based upon adding feed-forward signals [6,8,9] or adding feed-forward with inverse compensation filter [11]. This concept substantially reduces the axial tracking errors, and thereby reduces the contour error in linear cuts. A deficiency of this approach is that for perfect tracking it requires precise knowledge of the dynamic behavior of the axial drive system. However, this behavior is nonlinear and might vary with time, and therefore it is difficult to be modeled. Another drawback of this method is that reducing the axial errors does not necessarily reduce the contour error in nonlinear cuts. Consider, for example, the case in Fig. 1. Improvements in the axial control strategy shift the actual cutter location from point P to point P'. Although the axial errors $E_x'$ and $E_y'$ at point P' are smaller than $E_x$ and $E_y$, the contour error at P' is larger than that at P. (Further improvements will shift the cutter location to point P" and remedy the situation. However, this (1) requires case-by-case analysis, and (2) might cause instabilities.)

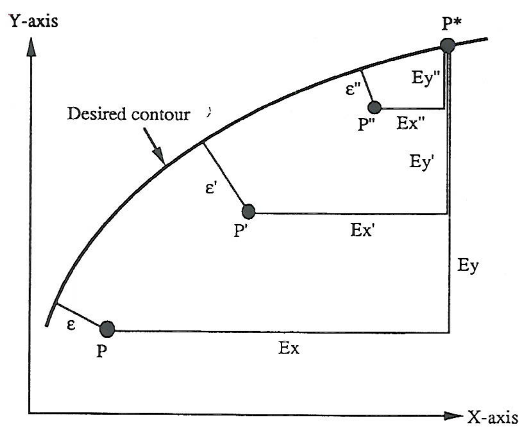


**Fig. 1** The axial and contour errors for different cutter locations.

The cross-coupling concept was introduced by Koren in [3]. The philosophy of the method is that the elimination of the contour error is the controller objective, rather than the reduction of the individual axial errors; in principle, a zero contour error may be achieved even with large axial errors. Therefore, the cross-coupling concept calls for the construction of a contour-error model in real time and utilizing it in the determination of a control law that reduce (or eliminate) the contour error.

A block diagram of a basic biaxial cross-coupling controller (CCC) is shown in Fig. 2. As can be seen, the axial position errors, $E_x$ and $E_y$, are used to determine the contour error, ε. The result is then multiplied by a proportional gain $W_p$ and added to each loop with the appropriate sign. The effects of the disturbances Dx and Dy as well as axis mismatch ( $K_x \neq K_y$ and $\tau_x \neq \tau_y$ ) are minimized with this controller. An improved cross-coupling controller was suggested in [4,5]. It applies the same concept of building a contour error model, but utilizes a more effective control law. In this case, the contour error is decomposed into two components which are then transferred through dynamic blocks $G_x$ and $G_y$ designed to compensate for the dynamic differences between the axes.

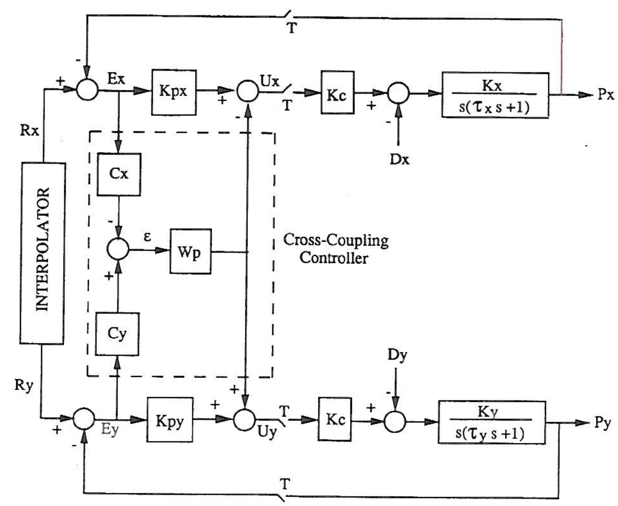


**Fig. 2** The basic cross-coupling controller for two axes.

The major drawback of the cross-coupling controllers presented in [3-5] is their low effectiveness in dealing with nonlinear contours such as circles and parabolas (the improved CCC in [5] provides better results, but these results are far from those presented below). Another drawback of these CCCs is that even for linear contours the contour error oscillates when the steady-state error goes to zero, which eliminates the complete cancellation of the contour error in linear cuts.

This paper introduces a new nonlinear CCC that is based on control gains that vary during the contour cut, and therefore it is called a Variable-Gain Cross-Coupling Controller. The gains are adjusted in real time according to the shape of contour. The variable-gain CCC eliminates the drawbacks mentioned above. Simulation results show the superiority of this new method.

## 2. The Variable-Gain Controller

The structure of the proposed variable-gain cross-coupling controller is shown in Fig. 3. The PID controller gains $W_p$, $W_i$, and $W_d$ are fixed for a particular system. The output of the PID controller is decomposed into two axial components by multiplying if by Cx and Cy. The axial components are then injected to the loops with the appropriate sign. this scheme ensures that contour error correction is executed in the proper direction. For a cut of linear segment the gains $C_x$ and $C_y$ are adjusted at the beginning of the segment. For nonlinear cuts they are adjusted continuously during the cut. The gains for the two cases are determined below.

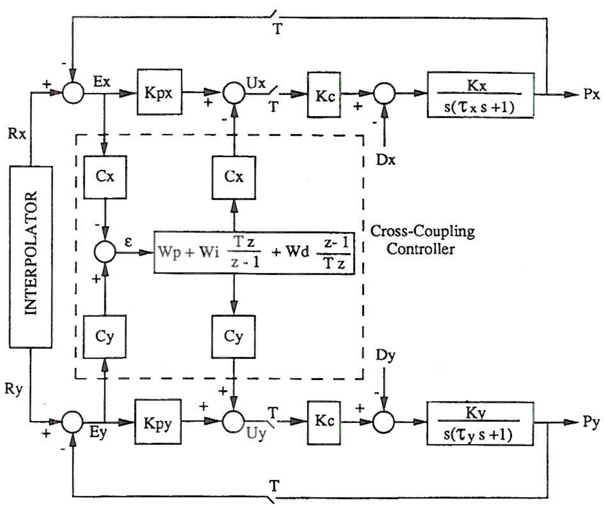


**Fig. 3** The variable-gain cross-coupling controller.

### 2.1 Linear Contour

The contour error can be determined from the geometrical relationship shown in Fig. 4.

$$\varepsilon = - E_x \sin\theta + E_y \cos\theta \tag{1}$$

Since $\sin\theta = V_y / V$ and $\cos\theta = V_x / V$, (V is the required feedrate), Eq. (1) yields

$$\varepsilon = \frac{- E_x V_y + E_y V_x}{V} \tag{2}$$

This equation has been introduced in [1]. Comparing the structure in Fig. 3 with Eq. (2) yields the cross-coupling gains

$$C_x = V_y / V \qquad C_y = V_x / V \tag{3}$$

The axial velocity components $V_x$ and $V_y$ depend on the slope of the linear cut, and they are adjusted at the beginning of each segment.

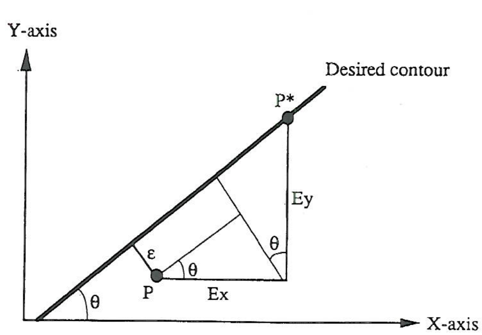


**Fig. 4** Error model for a linear contour.

### 2.2 Circular Contour

The contour error for a circular contour is the difference between the distance from the tool location to the center of the circle and the radius of the circle :

$$\varepsilon = \sqrt{(P_x - x_0)^2 + (P_y - y_0)^2} - R \tag{4}$$

Where R is the radius of the circle, $(x_0, y_0)$ is the corresponding center of the circle. The actual position $(P_x, P_y)$ can be represented by the axial errors and the reference position.

$$P_x = R_x - E_x = R \sin\theta + x_0 - E_x \tag{5}$$

$$P_y = R_y - E_y = - R \cos\theta + y_0 - E_y \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (4) yields

$$\varepsilon = \sqrt{(R \sin\theta - E_x)^2 + (- R \cos\theta - E_y)^2} - R \tag{7}$$

Because Eq. (7) is difficult to be implemented in a real-time control system, a simplification is proposed. Expanding the error in Eq. (7) by the Taylor series expansion yields

$$\varepsilon = - E_x \left( \sin\theta - \frac{E_x}{2R} \right) + E_y \left( \cos\theta + \frac{E_y}{2R} \right) + \text{Order} \left( \frac{\varepsilon^2}{R} \right) \tag{8}$$

If the contour error is much smaller than the axial errors, and the axial errors are much smaller than the radius of the circle, the high-order term can be neglected and the contour error can be approximated by :

$$\varepsilon = - E_x C_x + E_y C_y \tag{9}$$

Where $C_x$ and $C_y$ are defined by :

$$C_x = \sin\theta - \frac{E_x}{2R} \tag{10}$$

$$C_y = \cos\theta + \frac{E_y}{2R} \tag{11}$$

Note that R is constant and the gains are calculated at each interpolation step. In this calculation we use the interpolator input $\sin\theta$ and $\cos\theta$ as well as the axial error signals $E_x$ and $E_y$ that vary during the process. Eqs. (10) and (11) express the new concept of the variable gain.

### 2.3 General Nonlinear Contour

A non-circular contour is locally approximated by a circle as shown in Fig. 5. If the axial errors are much smaller than the instantaneous radius of curvature R, the contour error can appropriately be approximated by Eq. (7). Consequently, the variable gains are also given by Eqs. (10) and (11). The basic difference is that now the parameter R has to be calculated for each interpolation step, as well as $\sin\theta$ and $\cos\theta$. For example, for a parabolic shape $y = ax^2$ these parameters are

$$\sin\theta = \frac{2ax}{\sqrt{1 + (2ax)^2}} \tag{12}$$

$$\cos\theta = \frac{1}{\sqrt{1 + (2ax)^2}} \tag{13}$$

$$R = \frac{1}{2a} [1 + (2ax)^2]^{1.5} \tag{14}$$

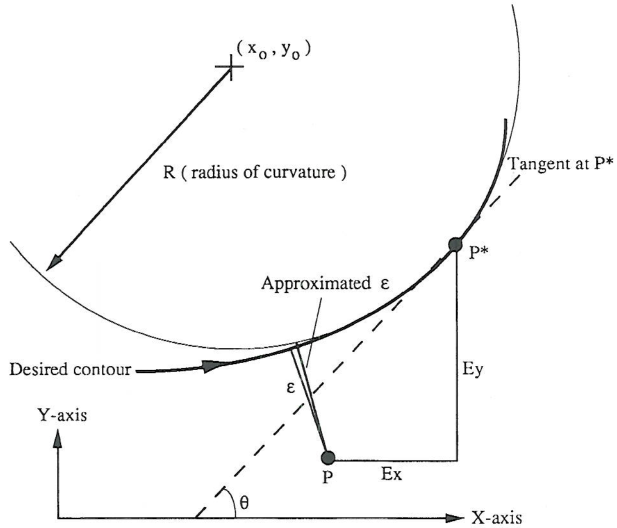


**Fig. 5** Error model for a nonlinear contour.

## 3. Simulation Results

A Comparison of the contour error between the variable-gain CCC and a conventional controller is shown in Figs. 6, 7, and 8 for the following data:

$K_cK_x = 10.3$ , $K_cK_y = 10.0$ , $\tau_x = 0.040$ , $\tau_y = 0.045$ , $K_{px} = K_{py} = 1.0$ ,
$W_p = 8.0$ , $W_i = 80.0$ , $W_d = 0.6$, $|Dx| = |Dy| = 0.75$ ,
Feedrate = 11.8 mm/sec ( 28 ipm ).

These parameters are similar to those on our experimental system. The disturbances are due to friction forces in the table guides.

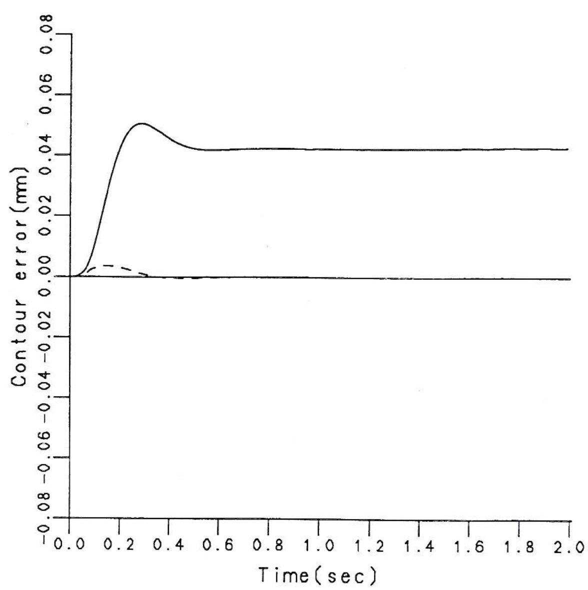


**Fig. 6** Simulation results for a 30° linear contour.
solid line : conventional control
dashed line : cross-coupling control

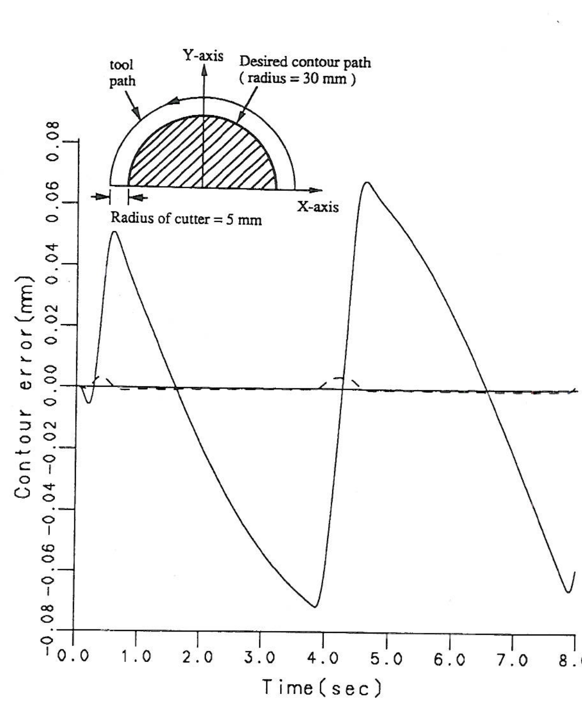


**Fig. 7** Simulation results for a circular contour.
solid line : conventional control
dashed line : cross-coupling control

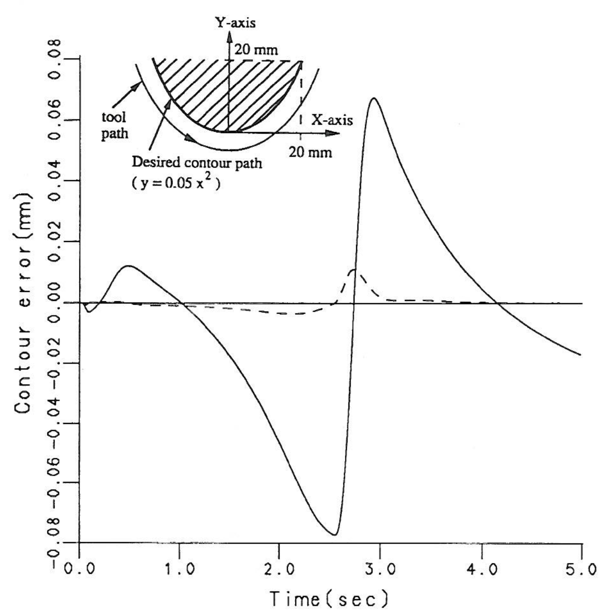


**Fig. 8** Simulation results for a parabolic contour.
solid line : conventional control
dashed line : cross-coupling control

The maximum contour errors of these simulation cases are summarized in the table below. The error reduction are significant.

Maximum Contour Error ( $\mu m$ )

| Contour type | Conventional ( P-controller ) | Cross-Coupling Control |
|---|---|---|
| Linear | 50.5 | 3.5 |
| Circular | 71.6 | 3.7 |
| Parabolic | 77.3 | 11.1 |

In a real system we do not expect this level of improvement because of the mechanical hardware deficiencies. However, error reduction of 3:1 to 10:1 can be achieved depending on the starting point and the resolution of the system.

## 4. Conclusions

A variable-gain cross-coupling controller that reduces the contour errors has been proposed. The method is based on building in real time the instantaneous contour error, feeding it into a PID controller, and decomposing the signal into axial components through multiplying it by gains that are calculated at each interpolation step. These variable gains have to be computed in real time, which obviously slows down the possible sampling-rate of the controller. However, with present advances in computer speeds, we don't see it as a drawback of the proposed controller, and we do believe that next generation CNC systems will be designed with cross-coupling controllers.

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