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Efficient Tool-Path Planning for Machining Free-Form Surfaces

This paper presents an analytical method for planning an efficient tool-path in machining free-form surfaces on 3-axis milling machines. This new approach uses a nonconstant offset of the previous tool-path, which guarantees the cutter moving in an unmachined area of the part surface and without redundant machining. The method comprises three steps: (1) the calculation of the tool-path interval, (2) the conversion from the path interval to the parametric interval, and (3) the synthesis of efficient tool-path planning.

1 Introduction

Free-form surfaces are widely used in CAD systems to describe the surfaces of parts, such as molds and dies. The surfaces are often produced by three-axis CNC machines using ball-end cutters. The current methods for machining those part surfaces require some important human decisions, such as determination of the precise interval between successive tool-paths. A tool-path interval that is too large can result in a rough surface; one that is too small can increase machining time, making the process inefficient. Another critical decision is to find efficient tool-paths of the entire part surface. The need for human decisions in the CAD/CNC process causes difficulties in the integration of the design with the manufacturing stages. To automate the design/manufacturing process, algorithms for efficient path planning based on accurate tool-path intervals are needed.

The usual method of machining free-form surfaces is to choose the small incremental isoparametric curves as the tool paths. The isoparametric curves, on a free-form surface $S(u, v)$, are obtained by keeping one parameter (either u or v) constant. An improvement of this method was suggested by Sata et al. (1981) and Kawabe et al. (1981) who chose, by experience, a small increment between two isoparametric curves as the tool-path interval. They did not, however, present a mathematical method to determine the tool-path interval. A mathematical method to obtain the tool-path interval for a flat plane was presented by Kato et al. (1984). This approach was expanded by Loney and Ozsoy (1987) for curved surfaces, but since they used the flat plane formula, their path intervals were inaccurate. In addition, their approach was limited to the surface with uniform parameterization based on arc length. Choi et al. (1988) planned the tool-paths on the xy-plane of a Cartesian coordinate, which was to find the intersection curves between the surface and vertical planes; however, this method can only choose the smallest increment for tool-path interval. Vickers and Quan (1989) expressed the scallop height as a function of the curvatures of convex or concave surfaces, the cutter radii, and the path intervals, but they did not present a procedure for selecting the tool-path interval.

Suh and Lee (1990) and Hwang (1992) presented a method to determinate the tool-paths by calculating, at each path, the smallest tool-path interval and using it as a constant offset in the next tool-path. The reason for their selecting the smallest interval as the offset distance is that this makes it easy to define the constant isoparametric offset as the next tool path, thereby satisfying the surface accuracy. However, one serious problem in this method is the nonpredictable scallop height remaining in the part surface, which causes either surface roughness (if too large) or inefficient machining (if too small). The inefficiency can be seen in Fig. 1. The region $ABCD$ represents the area that the first tool-path* $\overline{12}$ covers in the required surface accuracy (see Section 4 for details). If the next tool-path $\overline{34}$ is chosen by the smallest interval offset, this path covers the region $\overline{A'B'C'D'}$. Therefore, the dark area has been machined twice by the first two tool-paths; this is called redundant machining.

A tool-path planning method that kept the scallop height constant was presented by Suresh and Yang (1994). Their work led to a reduction of the CL data; however, reducing the CL data will not guarantee a proportional shortening of the machining time. In contrast, we adopt a new approach for determination of efficient tool-paths by a nonconstant parametric offset that results in smaller machining time than the conventional isoparametric tool-paths. For simplicity, an isoparametric boundary curve of the free-form surface is chosen for the first tool-path. However, any of the curves lying on the surface can be selected as the first tool-path. Along this path, a nonconstant offset curve, path $\overline{35}$ in Fig. 1, can be found by calculating its path intervals based on the constant scallop height. The machined area between this curve and the previous path maintains a constant scallop; therefore, this curve is chosen to be the next tool-path, which guarantees that it has no redundant motion.

To implement this approach, the precise tool-path interval has to be determined. Suresh and Yang (1994) expressed the tool-path interval in terms of the cutter radius, the scallop height, and the radius of curvature. It resulted in a very complicated expression of the tool-path interval and required a tedious iterative solution process, as compared to our neat solution for the tool-path interval. Our method is based on the derivation of scallop height (Vickers and Quan, 1989). However, the determined tool-path interval is not consistent with the parametric interval, and a conversion method is needed. A conversion method using the first-order Taylor expansion was presented by Suresh and Yang (1994), but it is only valid when the converted parametric interval is very small. The conversion technique that is introduced here is based on both the second order Taylor expansion and an error compensation method, and results in very accurate conversion between the tool-path interval and the parametric interval. Finally, a synthetic procedure for the planning of the efficient tool-paths for free-form surfaces is presented.

2 The Tool-Path Interval

The tool path in milling is the trajectory of the cutter center, which is also known as the *cutter location* (CL) path, and is given by the NC part program. The *cutter-contact* (CC) path is the tangential trajectory between the ball-end cutter and the part surface, and is shifted from the tool path by a distance

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* The path $\overline{12}$ (or $\overline{34}$) is defined here as the tool-path; the path $\overline{23}$ is defined as the tool-path interval.

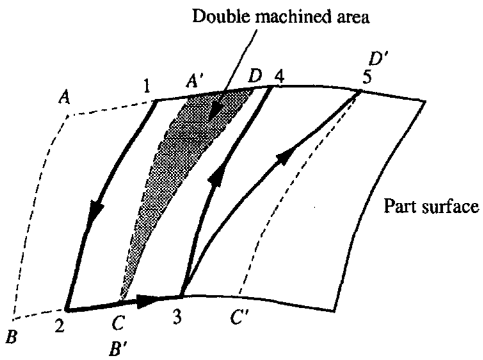

Fig. 1 The isoparametric tool-paths cause redundant machining

equal to the cutter radius (see Fig. 2). When the cutter moves in parallel trajectories, as shown in Fig. 2, scallops are created on the finished surface. The distance between the parallel trajectories is the CC *path interval,* which depends on the local curvature of the surface, the size of the cutter, and the allowable scallop height remaining on the surface after the machining operation. By demanding that the scallop height remains at a given constant value, the CC path interval can be determined as the distance that the cutter can slide without exceeding the allowable surface finish value. The *tool-path interval* can then be obtained by offsetting the CC path in the surface normal direction by a distance equal to the cutter radius, a procedure which is described in Section 4.

In this section, the mathematical formula of the tool-path interval will be derived as a function of the cutter radius, the scallop height, and the radius of surface curvature. The cutter radius and the scallop height are given constants, but the radius of surface curvature is changing during machining. The derivation of the radius of surface curvature is explained below. To obtain the precise tool-path interval, the direction of the tool-path interval has to be orthogonal to the tool-path itself. The radius of curvature, in this direction, of a surface $S(u, v)$ can be obtained by computing the first and the second fundamental matrices, G and D respectively, of the surface (Faux, 1981),

$$R = \frac{\dot{w}^T G \dot{w}}{\dot{w}^T D \dot{w}} \tag{1}$$

where $\dot{w} = [\dot{u}\dot{v}]$ and

$$G = \begin{bmatrix} \dfrac{\partial S}{\partial u}\cdot\dfrac{\partial S}{\partial u} & \dfrac{\partial S}{\partial u}\cdot\dfrac{\partial S}{\partial v} \\[2ex] \dfrac{\partial S}{\partial v}\cdot\dfrac{\partial S}{\partial u} & \dfrac{\partial S}{\partial v}\cdot\dfrac{\partial S}{\partial v} \end{bmatrix} \quad D = \begin{bmatrix} \mathbf{n}\cdot\dfrac{\partial^2 S}{\partial u^2} & \mathbf{n}\cdot\dfrac{\partial^2 S}{\partial u \partial v} \\[2ex] \mathbf{n}\cdot\dfrac{\partial^2 S}{\partial v \partial u} & \mathbf{n}\cdot\dfrac{\partial^2 S}{\partial v^2} \end{bmatrix}$$

R is the radius of curvature in the orthogonal direction and $\mathbf{n}$ is the unit normal vector to the surface. Equation (1) involves two unknowns, $\dot{u}$ and $\dot{v}$, which can be solved as following. Let us define a plane that is orthogonal to the tool-path and parallel to the surface normal. The intersection between this plane and the surface is a curve F which is lying on the surface and orthogonal to the tool-path. Since the tangent of F is also orthogonal to the tangent of the tool-path, we get,

$$\dot{F}^T \mathbf{T} = 0 \tag{2}$$

where $\mathbf{T}$ is the tangent vector of the tool-path and $\dot{F} = (\partial F/\partial u)\dot{u} + (\partial F/\partial v)\dot{v}$. The ratio $\dot{u}:\dot{v}$ can be calculated from Eq. (2)

$$\frac{\dot{u}}{\dot{v}} = \frac{\dfrac{\partial F}{\partial v}\cdot\mathbf{T}}{-\dfrac{\partial F}{\partial u}\cdot\mathbf{T}} \tag{3}$$

Therefore, the radius of surface curvature R is orthogonal to the tool-path and can be determined by substituting Eq. (3) into Eq. (1).

In the following subsections, the geometric relationships when machining a flat plane, a convex surface, and a concave surface with a ball-end cutter are developed. In each of these three cases, the tool-path interval is derived. For the flat-plane case, a closed analytical solution of the CC path interval can be easily obtained (Loney and Ozsoy, 1987). For the curved surfaces, however, the solution is more complex, and approximation methods are also considered.

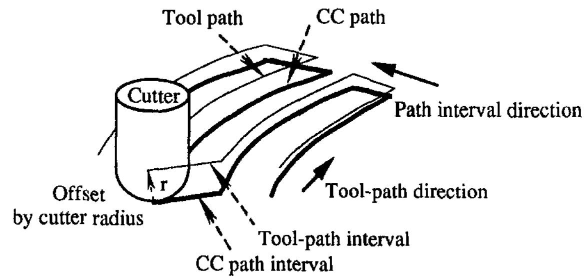

Fig. 2 Illustration of the tool path, CC path, path interval, and cutting direction

2.1 Flat Planes Machined by a Ball-End Cutter. The calculation of the CC path interval for milling flat planes is relatively simple. When the milling operation is completed, scallops remain on the finished surface as shown in Fig. 3. For a given allowable scallop height the CC path interval can be obtained by using the Pythagoras theorem (Loney and Ozsoy, 1987):

$$P = 2\sqrt{r^2 - (r-h)^2} \cong 2\sqrt{2rh} \tag{4}$$

where P denotes the tool-path interval, r denotes the radius of a ball-end cutter, and h denotes the allowable scallop height. The assumption in the approximation of Eq. (4) is that $h \ll r$, which is reasonable since typically $r > 1$ mm and $h < 0.01$ mm (depending on the surface finish requirement). Equation (4) is an analytical solution of the CC path interval in flat-plane machining. In this case, the tool-path interval P_{cl} is equal to the CC path interval P since the surface normal is in a uniform direction.

2.2 Convex Surfaces Machined by a Ball-End Cutter. The calculation of the maximum allowable path interval for a general convex surface is more complicated than for a flat surface. A convex surface machined by a ball-end cutter is shown in Fig. 4. The CC path interval depends on the curvature of the surface, the size of the cutter, and the allowable scallop height remaining on the surface. As shown in Appendix A, the scallop height can be derived in terms of the path interval, the cutter radius, and the local radius of curvature of the convex surface.

$$h = (R+r)\sqrt{1 - \left(\frac{P}{2R}\right)^2} - \sqrt{r^2 - \left[\frac{(R+r)P}{2R}\right]^2} - R \tag{5}$$

where R is the local radius of curvature of the convex surface.

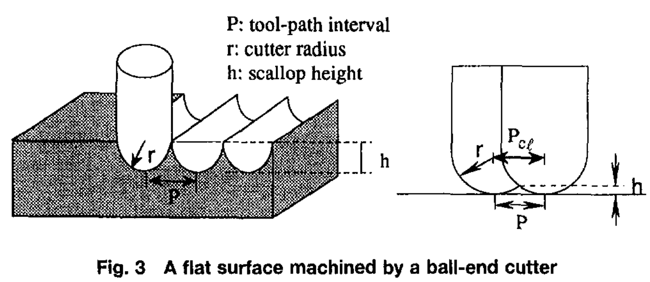

Fig. 3 A flat surface machined by a ball-end cutter

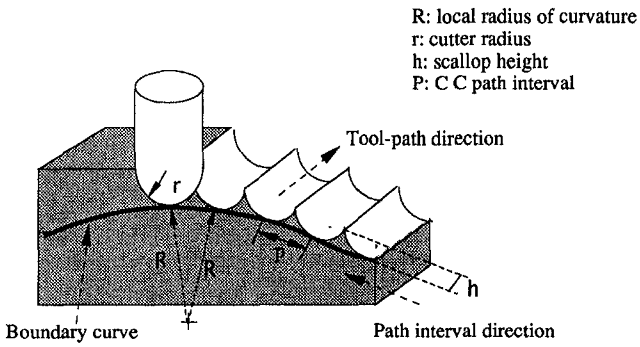

Fig. 4 A convex surface machined by a ball-end cutter

The method for obtaining an explicit formula for the CC path interval, $P = P(R, r, h)$, is presented below. Equation (5) has the format of

$$c = \sqrt{a} - \sqrt{b}. \tag{6}$$

where

$$a = (R + r)^2\left[1 - \left(\frac{P}{2R}\right)^2\right],$$

$$b = r^2 - \left[\frac{(R + r)P}{2R}\right]^2, \quad \text{and} \quad c = h + R.$$

Using some mathematical manipulations, Eq. (6) can be written as

$$(a - b)^2 + c^4 = 2(a + b)c^2 \tag{7}$$

Substituting the values of a, b, and c in Eq. (7) and defining $q = R + r$ yields the formula for the calculation of the tool-path interval

$$P^2 = \left[\frac{R}{(R + h)q}\right]^2 [2(q^2 + r^2)(R + h)^2 - (q^2 - r^2)^2 - (R + h)^4] \tag{8}$$

Since in practice $R \gg h$, Eq. (8) may be approximated by substituting

$$(R + h)^2 = R^2 + 2Rh \quad \text{and} \quad (R + h)^4 = R^4 + 4R^3h$$

in the numerator and $R + h = R$ in the denominator of Eq. (8), which yields the approximated solution of the CC path interval

$$P = \sqrt{\frac{8hrR}{R + r}} \tag{9}$$

The CC path intervals corresponding to the individual cutter radii and radii of curvatures of the part surface for a constant h as calculated by Eq. (9) are shown in Fig. 5. Again, the tool-path interval can be obtained by offsetting the CC path interval.

Utilizing the approximation in Eq. (9) presents a trade-off between precision and calculation time. An evaluation method of the precision level of the approximation is presented below. By substituting the solution of Eq. (9) into (8), the actual scallop height that remains on the surface can be obtained and is denoted as h'. The accuracy of Eq. (9) can be examined by the error $\Delta h = h' - h$, where h is the desired allowable scallop height. A 3-dimensional representation of the errors in terms of cutter radii and the radii of surface curvatures for a constant h is shown in Fig. 6. The conclusion is that for cutters with $r > 1$ mm and surfaces with $R > 1$ mm the approximated method can be applied if the requirement $\Delta h < 0.0002$ mm is satisfactory. That means that Eq. (9) can be used for most practical cases.

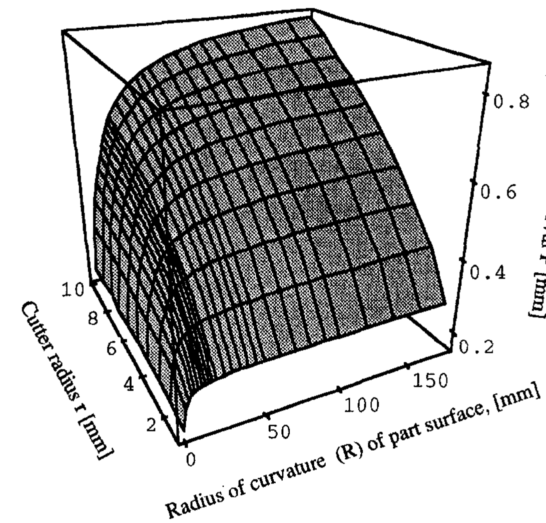

Fig. 5 The tool-path interval on convex surfaces based on a constant scallop height, $h = 0.01$ mm

2.3 Concave Surfaces Machined by a Ball-End Cutter. The CC path interval in machining a concave surface can be derived in a way similar to the convex case. Figure 7 illustrates a concave surface machined by a ball-end cutter. The scallop height (see Appendix B) is given by

$$h = R - (R - r)\sqrt{1 - \left(\frac{P}{2R}\right)^2} - \sqrt{r^2 - \left[\frac{(R - r)P}{2R}\right]^2} \tag{10}$$

The maximum CC path interval, obtained by the method shown above, is

$$P^2 = \left[\frac{R}{(R - h)q}\right]^2 [2(q^2 + r^2)(R - h)^2 - (q^2 - r^2)^2 - (R - h)^4] \tag{11}$$

where $q = R - r$.

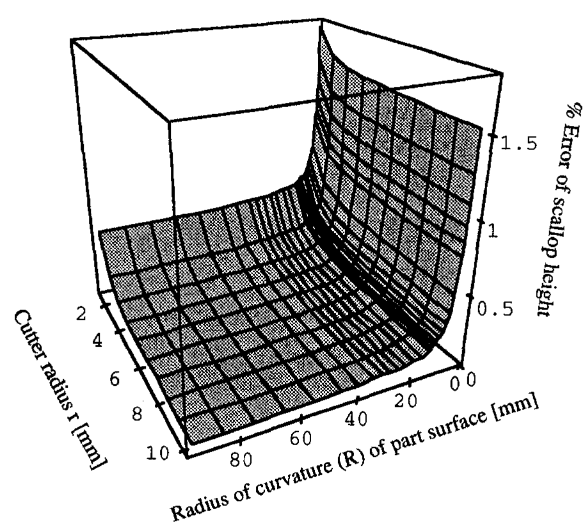

Fig. 6 The percentage error of the scallop height caused by using approximation in convex surface machining

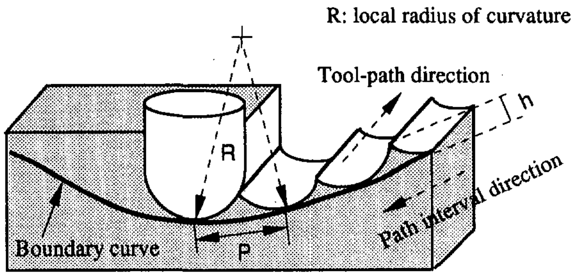

Fig. 7 Concave surface machined by a ball-end cutter

The corresponding approximated CC tool-path interval is

$$P = \sqrt{\frac{8hrR}{R - r}} \tag{12}$$

Figure 8 shows the CC path interval corresponding to the individual cutter radii and radii of surface curvatures for a constant h as calculated by Eq. (12).

Equations (11) and (12) are valid when the cutter radius is smaller than the radius of surface curvature, $r < R$, which is the typical case in machining concave surfaces. Note that some singular points exist when the radius of surface curvature is equal to the cutter radius, i.e., $R = r$. In the case where the cutter radius is larger than the radius of surface curvature, $r > R$, the cutter will gouge the part surface, which causes an overcut. Figure 9 shows the error profiles resulting when the proposed approximated methods are used. It also shows some protrusions near the singular points. Again, the approximation can be used for $\Delta h < 5 \times 10^{-5}$ mm (if the cutter radius is smaller than the radius of surface curvature, $r < R$), which fits most practical cases.

A flat surface is a particular case of a convex surface with an infinite radius of curvature, i.e., $R \rightarrow \infty$. For the flat surface case, R in the numerator and $(R + r)$ in the denominator of Eq. (9) are canceled out, and Eq. (9) becomes identical to Eq. (4), which is for the flat-surface case. The same analysis is also applied to Eq. (12), the concave surface case.

3 Conversion of the CC Path Interval to the Parametric Domain

The determined tool-path interval in the orthogonal direction cannot be directly adopted in parametric domains because: (*1*) the direction of the path interval is not located in the same direction as the parametric curve (See Fig. 10), and (*2*) the path interval is calculated in distance units (mm or inch) which do not match the unit of the parametric interval. Therefore, the tool-path interval in the orthogonal direction has to transfer to the parametric direction by,

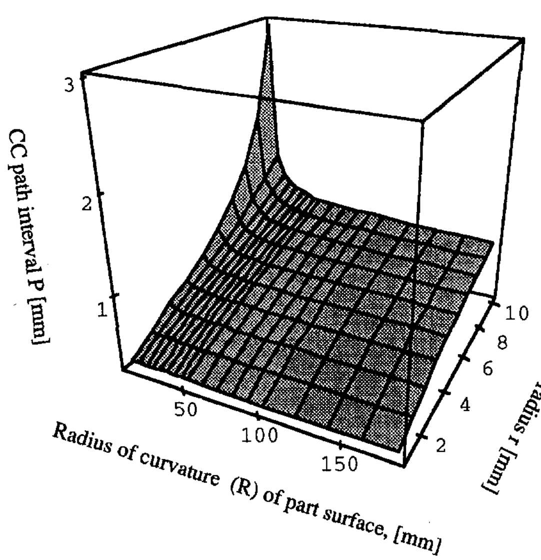

Fig. 8 The tool-path interval on concave surfaces based on a constant scallop height, h = 0.01 mm

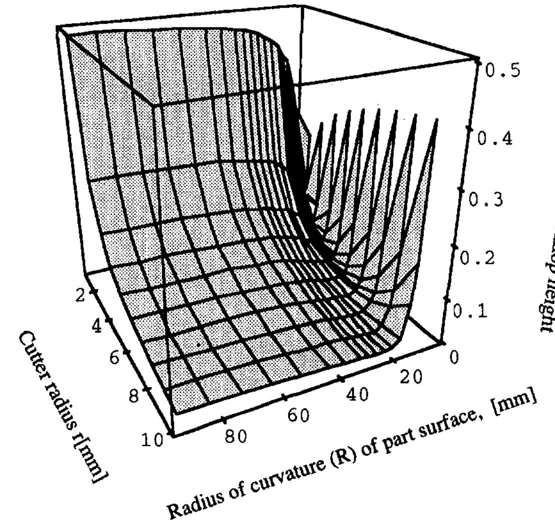

Fig. 9 The percentage error of the scallop height caused by using approximation in concave surface machining

$$P_p = \frac{P}{\sin \theta} \tag{13}$$

where P_p is the path interval in the parametric direction and P is the path interval in the orthogonal direction. θ is the angle between the tool-path and the parametric curve C which can be calculated by

$$\theta = \cos^{-1}\left(\frac{\frac{\partial C}{\partial u}\cdot(-\mathbf{T})}{\left|\frac{\partial C}{\partial u}\right|\cdot|-\mathbf{T}|}\right) \tag{14}$$

where $\mathbf{T}$ is the tangent vector of the tool-path and $\partial C/\partial u$ is the tangent of the parametric curve.

Next, a conversion from the physical domain P_p to the parametric domain Δu is needed in order to place the tool on the calculated path. This conversion, which uses the Taylor expansion and an error compensation technique to speed up the solution, is presented below.

Given a parametric curve $\vec{C}(u)$, $0 \le u \le 1$, which is used to determine the CC path interval, the Taylor series expansion of this parametric curve is

$$\vec{C}(u) = \vec{C}(u_0) + \vec{C}'(u_0)\Delta u + \frac{1}{2}\vec{C}''(u_0)\Delta u^2 + \frac{1}{3!}\vec{C}'''(u_0)\Delta u^3 + \ldots \tag{15}$$

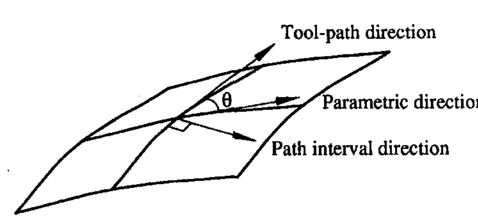

Fig. 10 The angular difference θ between the path-interval and the parametric directions

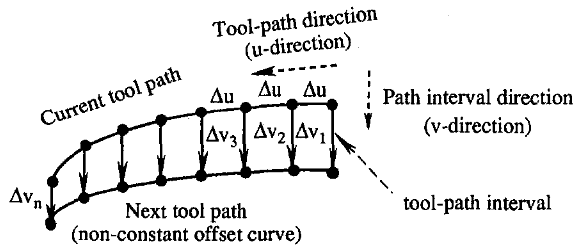

Fig. 11 Illustration of finding the next efficient path based on accurate tool-path intervals

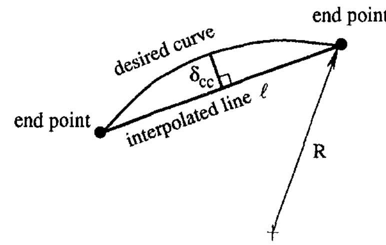

Fig. 13 Linear approximation of a desired curve

where

$$0 \leq u \leq 1, \quad \vec{C}(u) = x(u)\hat{i} + y(u)\hat{j} + z(u)\hat{k},$$
$$\text{and} \quad \Delta u = u - u_0.$$

Δu is the incremental quantity in the parametric domain that corresponds to the CC path interval in the physical domain, and u_0 is the current tool contact position in the parametric domain. The term $|\vec{C}(u) - \vec{C}(u_0)|$ is actually the CC path interval in the parametric direction, P_p. If we neglect the higher order terms, the path interval can be derived as below,

$$P_p = |\vec{C}(u) - \vec{C}(u_0)| = |\vec{C}'(u_0)\Delta u + \tfrac{1}{2}\vec{C}''(u_0)\Delta u^2| \quad (16)$$

or,

$$P_p^2 = A\Delta u^4 + B\Delta u^3 + D\Delta u^2 \quad (17)$$

where,

$$\begin{cases} A = \dfrac{1}{4}\left[\left(\dfrac{d^2x}{du^2}\right)^2 + \left(\dfrac{d^2y}{du^2}\right)^2 + \left(\dfrac{d^2z}{du^2}\right)^2\right]_{u=u_0} \\ B = \left[\dfrac{dx}{du}\cdot\dfrac{d^2x}{du^2} + \dfrac{dy}{du}\cdot\dfrac{d^2y}{du^2} + \dfrac{dz}{du}\cdot\dfrac{d^2z}{du^2}\right]_{u=u_0} \\ D = \left[\left(\dfrac{dx}{du}\right)^2 + \left(\dfrac{dy}{du}\right)^2 + \left(\dfrac{dz}{du}\right)^2\right]_{u=u_0} \end{cases}$$

Equation (17) which represents the CC path interval P_p, is a highly nonlinear function of the parametric interval (Δu) and requires a tedious iterative solution process and a proper initial value such as Newton's method. To speed up the solution process, we introduce an error-compensation method to solve Δu in Eq. (17).

The first order approximation of Eq. (17) is

$$\Delta u_a = \frac{P_p}{\sqrt{\left[\left(\dfrac{dx}{du}\right)^2 + \left(\dfrac{dy}{du}\right)^2 + \left(\dfrac{dz}{du}\right)^2\right]_{u=u_0}}} \quad (18)$$

The above equation is valid if Δu is very small, namely, when u and u_0 are very close. By comparing Eq. (17) and (18), an error indicator can be defined as

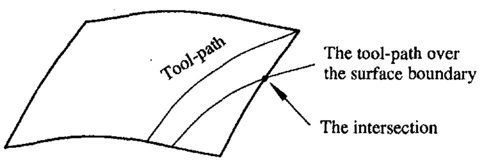

Fig. 12 The intersection of the surface boundary and the tool-path over the end parameter

$$\epsilon = A\Delta u^4 + B\Delta u^3. \quad (19)$$

The error indicator in Eq. (19) is calculated by using $\Delta u = \Delta u_a$ from Eq. (18) and is then substituted back into Eq. (17) to obtain a more accurate conversion from the CC path interval to the parametric interval

$$\Delta u = \frac{\sqrt{P_p^2 - \epsilon}}{\sqrt{\left[\left(\dfrac{dx}{du}\right)^2 + \left(\dfrac{dy}{du}\right)^2 + \left(\dfrac{dz}{du}\right)^2\right]_{u=u_0}}} \quad (20)$$

By using this error-compensation method, the accuracy is at the same order of magnitude as Newton's method, and only one iteration is needed. This first-order error-compensation method can convert the path interval into the parametric interval for the machine tools with an accuracy of up to 10^{-4} mm, which is more than adequate for most systems.

4 Efficient Tool-Path Planning

The CC path interval, which depends on the surface curvature, is the distance that a cutter can slide over the surface without losing accuracy. For a flat-plane surface these intervals are constant because the surface curvature is zero everywhere, and the next tool-path is just the constant offset of the previous tool-path. In contrast, the curvatures on a general 3-D surface are not constant; therefore, using the constant offset method causes either inaccurate or inefficient machining. In this section, a new approach for determination of the efficient tool-paths by a nonconstant offset is presented that results in less machining time than the conventional isoparametric tool-paths.

4.1 Searching for the Efficient Tool-Path. For simplicity, an isoparametric boundary curve of the free-form surface is chosen as the beginning tool-path. However, any of the curves lying on the surface can be selected as the first tool-path. Along this path, a nonconstant offset curve can be found by calculating its path intervals based on the constant scallop height (Fig. 11). In numerical computation, these intervals are stored in an array. The accumulated arrays represent this nonconstant offset curve in the parametric domain. Therefore, this curve is the next efficient tool-path which guarantees that the tool-path will maintain the constant scallops on the part surface and that the tool-path will not have redundant motion (path 35 in Fig. 1). We call this an efficient machining.

The searching for efficient tool-paths stops when all the elements of the accumulated array, the accumulated path-intervals in the parametric domain, are greater than the end-parameter, normally $0 \leq u, v \leq 1$. In some cases during the search (Fig. 12) where some values of the accumulated path-intervals are greater than the end-parameter and some are not, then the intersection of the tool-path and the boundary curve has to be determined as the beginning or the ending point of the path. This makes the tool-path begin and end at the surface boundary.

In 3-axis machining with ball-end cutters, the tool-path is an offset of the CC path by a distance equal to the cutter radius

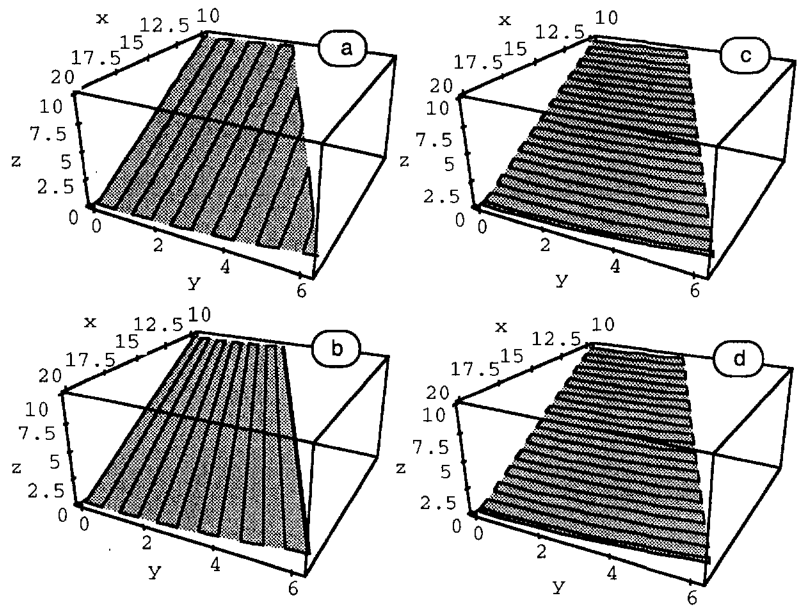

Fig. 14 **(*a*) Efficient tool-paths in *v*-direction (*b*) isoparametric tool-path in *v*-direction (*c*) efficient tool-paths in *u*-direction (*d*) isoparametric tool-path in *u*-direction**

with the orientation of the surface normal (Faux, 1981). The offset path from the CC path is calculated by

$$S_{cl}(u, v) = S_{cc}(u, v) + r \cdot \frac{\dfrac{\partial S_{cc}}{\partial u} \times \dfrac{\partial S_{cc}}{\partial v}}{\left| \dfrac{\partial S_{cc}}{\partial u} \times \dfrac{\partial S_{cc}}{\partial v} \right|} \tag{21}$$

where S_{cc} is the CC path, S_{cl} is the CL path, and r is the cutter radius.

In some cases, the offset path will interfere with the original path, which causes overcut problems. For example, the offset path will cause surface gouging when the local radius of curvature of a concave curve is smaller than the cutter radius. Wysocki et al. (1989) and Choi and Jun (1989) investigated those problems and presented some techniques to avoid gouging while offsetting the CC path. Those methods can be utilized to offset the obtained CC path to a gouge-free tool path.

The efficient tool paths discussed so far are represented by a set of 3-dimensional data-points. However, there is no guarantee that these tool paths are any one order of mathematical form or any one family curve of the original surfaces. Therefore, a curve-fitting technique is needed to machine efficiently along the tool path. The simplest curve fitting utilizes the first order approximation (Faux, 1981). For machine tools that have the capability to cut higher order curves, a higher order curve fitting can be adopted to interpolate the tool-paths for more efficient machining. In this paper, the first order approximation is used, since the linear motion is a fundamental function of all numerical control machine tools.

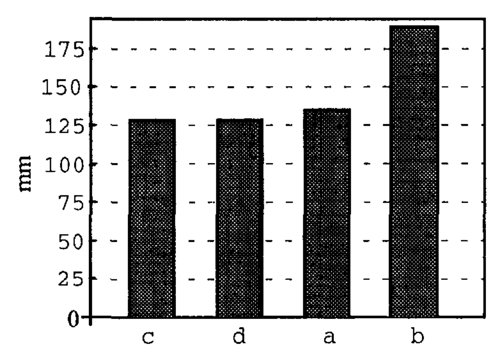

Fig. 15 **The total path lengths of four kinds of tool-paths**

We start by fitting the CC path, and then modify it to fit the tool path (CL path). Shown in Fig. 13 is the linear approximation of a desired curve (CC path). The subscript *cc* stands for a curve of a CC path. The critical equation to determine the linear approximation by the recursive method (see Appendix C) is given by

$$\delta_{cc} = \frac{l^2}{8R} \tag{22}$$

where R is the radius of curvature of the desired curve, l is the length of the straight-line approximation, and δ_{cc} is the maximum allowable deviation when using the linear approximation. To fit a tool-path that has an offset from the CC path, the above equation needs to be modified. For a convex case it is modified to (see Appendix C)

$$\delta_{cl} = \left(\frac{R + r}{R}\right)\delta_{cc} \tag{23}$$

and for a concave case to

$$\delta_{cl} = \left(\frac{R - r}{R}\right)\delta_{cc} \tag{24}$$

Equation (23) and (24) provide the maximum allowable deviation for linearly approximating the tool-path with the minimum number of straight-lines while still maintaining the required surface finish.

4.2 The Procedure of Finding the Efficient Tool-Path. A summary of the efficient tool-path determination procedure is given below.

(*1*) Choose either one of the surface parameters (say v) as the tool-path direction. Thus the first isoparametric curve at $u = 0$ is the first CC path.

(*2*) Offset the CC path to the CL path by Eq. (21).

(3) Curve-fit the CL path, which is the first tool path.
(4) Compute the CC path intervals in the direction of the tool-path normal by increasing at a small interval, (Δv) along the first CC path (Fig. 11). The equation for calculating the accurate CC path interval is given in Section 2.
(5) Convert the CC path interval into the u-domain by Eqs. (13) and (20), also store the u-values in an array corresponding to the v-values.
(6) The profile of these u and v values is the next efficient CC path.
(7) Offset this CC path to a CL path.
(8) Curve-fit the CL path, which is the next efficient tool path.
(9) The searching for the next path stops when the accumulated u's reach the end point, $u = 1$ (see Section 4.1).

Following above procedure, a set of tool-paths for the entire surface that does not violate the h constraint can be obtained.

5 Examples

In this section, two examples are given to illustrate the placement of efficient tool-paths based on accurate path intervals. The cutting efficiency of our method is compared with that of the isoparametric method presented by Wysocki et al. (1989) and Choi et al. (1987).

Example 1: A part surface is described by a portion of a cone surface to be machined by a 3-axis NC milling machining with a ball-end cutter. The equations for this surface are shown below:

$$\begin{cases} X = [10(1 - v) + 20v] \cos(u) \\ Y = [10(1 - v) + 20v] \sin(u) \\ Z = 20(1 - v) + 10v \end{cases}$$

$$\text{where} \quad 0 \le v \le 1 \quad \text{and} \quad 0 \le v \le \frac{\pi}{10}.$$

The allowable scallop height is 0.01 mm and the cutter radius is 5 mm. By following the procedures described in Section 4, the efficient CC paths and the isoparametric CC paths can be determined in both u and v directions, which are shown in Fig. 14. In the v-direction, the total length of the efficient paths is 135.7 mm; it is 188.6 mm for conventional isoparametric paths. Figure 15 shows the total lengths of these paths. Therefore, the isoparametric path in the v-direction has 39 percent more cutting length than that of the efficient path in the same direction. To estimate the cutting time, the constant feedrate machining is assumed to have the total cutting time proportional to the total cutting length. Therefore, the isoparametric tool-paths cause 39 percent of redundant machining in this direction.

In the u-direction these two kinds of paths are identical and have the same cutting efficiency, 128.6 mm, since the path intervals are constant. This situation also happens when the part surface is a developable surface, which is generated by a straight line as it moves parallel to itself along a curve (Mortenson, 1985).

Example 2: A bi-cubic free-form surface with the equations are shown below:

$$\begin{cases} X = \begin{pmatrix} 10 + 10u - 15u^2 + 10u^3 + 10v - 60u^2v + 40u^3v \\ - 75v^2 + 60uv^2 + 360u^2v^2 - 240u^3v^2 + 50v^3 \\ - 40uv^3 - 240u^2v^3 + 160u^3v^3 \end{pmatrix} \\ Y = \begin{pmatrix} 20 + 15u^2 - 10u^3 - 10v - 30u^2v + 20u^3v \\ + 15v^2 - 30uv^2 - 90u^2v^2 + 60u^3v^2 - 10v^3 \\ + 20uv^3 + 60u^2v^3 - 40u^3v^3 \end{pmatrix} \\ Z = \begin{pmatrix} 15u - 15u^2 + 20v - 20v^2 + 45uv^2 \\ - 45u^2v^2 - 30uv^3 + 30u^2v^3 \end{pmatrix} \end{cases}$$

where $0 \le u, v \le .2$.

The machining condition is the same as example 1. Figure 16 shows two kinds of tool paths in the u and v directions. In v-direction, the total length of the efficient paths is 29.8 mm, and it is 34.8 mm for isoparametric paths. In the other direction, these are 33.1 mm and 36.8 mm for efficient paths and isopara-

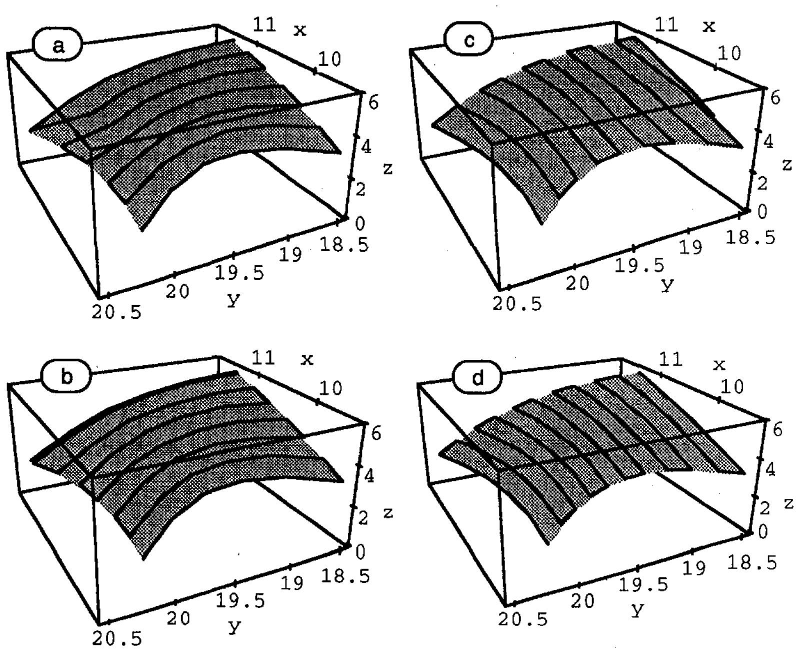

Fig. 16 (*a*) Efficient tool-paths in *v*-direction (*b*) isoparametric tool-path in *v*-direction (*c*) efficient tool-paths in *u*-direction (*d*) isoparametric tool-path in *u*-direction

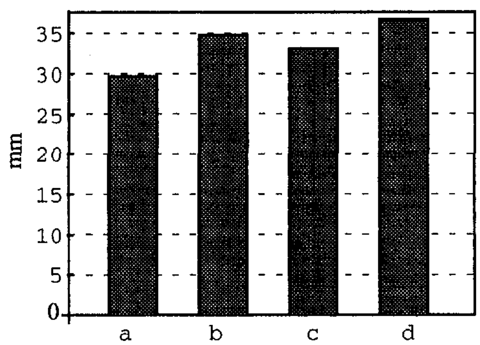

Fig. 17 The total pathlengths of four kinds of tool-paths

metric paths, respectively. Figure 17 shows the total lengths of these paths. From these cutting lengths, the isoparametric paths in the v-direction cause 17 percent of inefficient machining compared with the efficient paths in the same direction, and 11 percent of inefficient machining compared with the efficient paths in the u-direction.

6 Conclusion

Example 1 demonstrated that the cut-length efficiency of the proposed method improves 39 percent compared with the conventional isoparametric method. The efficiency depends highly on the local geometric properties, such as the curvature and the parameterization, of the part surfaces. In some cases, such as developable surfaces, the efficiency of the efficient paths will be the same as with the isoparametric paths. The reason for obtaining the same efficiencies is that the path intervals are constant between the two paths. It turns out that the determined efficient paths are the same as the conventional isoparametric paths on the developable surface.

The scallops remaining on the finished part surface are not constant by isoparametric path machining because a constant interval offset is chosen for the next isoparametric path. In contrast, the efficient path machining produces a part surface with a constant scallop.

As shown in the previous examples and derivations, this algorithm for efficient tool-path planning is not limited in any special form or by any order of a parametric surface. Any of the parametric surfaces with C^2 continuity can have the efficient tool-path planning.

7 Summary

An analytical study of a new algorithm for determination of the efficient tool-path of a free-form surface is presented in this paper. This approach uses a nonconstant offset of the previous tool-path, which guarantees the cutter moving in an unmachined area of the part surface and without redundant machining. This research is also comprised of the investigation of the accurate tool-path interval and its conversion into the parametric interval.

The derivation of the maximum CC path interval in this paper is based on a ball-end cutter in a 3-axis machine tool. However, the methodology can also be applied to use a flat-end cutter for convex surfaces in 5-axis machine tools.

Acknowledgment

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APPENDIX A

From Fig. A, the equation of the circle can be obtained:

$$(x - q\cos\theta)^2 + (y - q\sin\theta)^2 = r^2$$

where

$$\cos\theta = \frac{\sqrt{R^2 - \left(\frac{P}{2}\right)^2}}{R}, \quad \sin\theta = \frac{P}{2R}, \quad \text{and} \quad q = R + r.$$

In order to find the intersection of the circle and the X-axis, we have to solve the simultaneous equations:

$$\begin{cases} \left(x - \frac{q}{R}\sqrt{R^2 - \left(\frac{P}{2}\right)^2}\right)^2 + \left(y - q\frac{P}{2R}\right)^2 = r^2 \\ y = 0 \end{cases}$$

We solve for x and choose the smaller root because of the physical sense from Fig. A.

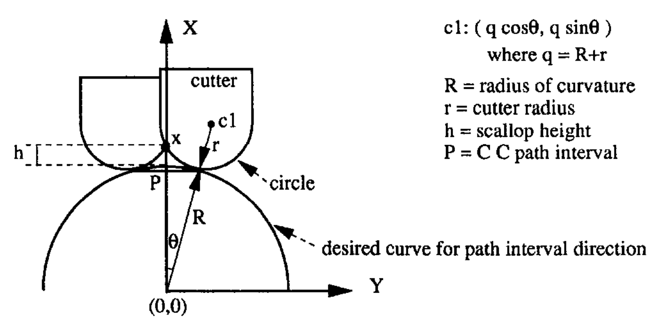

Fig. A Illustration of derivation of Eq. (5). The cutting path direction is into/out of the paper.

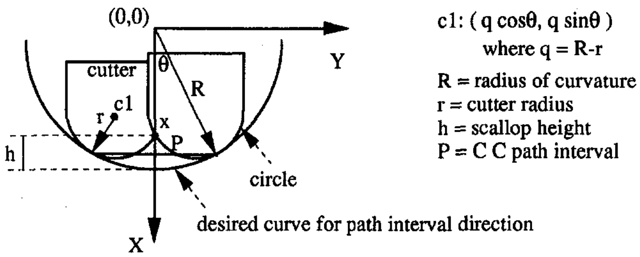

Fig. B Illustration of derivation of Eq. (10). The cutting path direction is into/out of the paper.

$$\Rightarrow x = \frac{q}{R}\sqrt{R^2 - \left(\frac{P}{2}\right)^2} - \sqrt{r^2 - \left(\frac{qP}{2R}\right)^2}$$

From Fig. A, we get $h = x - R$,

$$h = (R + r)\sqrt{1 - \left(\frac{P}{2R}\right)^2} - \sqrt{r^2 - \left(\frac{(R + r)P}{2R}\right)^2} - R \quad (5)$$

APPENDIX B

From Fig. B, the equation of the circle can be obtained:

$$\left(x - \frac{q}{R}\sqrt{R^2 - \left(\frac{P}{2}\right)^2}\right)^2 + \left(y - q\frac{P}{2R}\right)^2 = r^2$$

where $q = R - r$.

In order to find the intersection of the circle and the X-axis, we have to solve the simultaneous equations:

$$\begin{cases} \left(x - \frac{q}{R}\sqrt{R^2 - \left(\frac{P}{2}\right)^2}\right)^2 + \left(y - q\frac{P}{2R}\right)^2 = r^2 \\ y = 0 \end{cases}$$

We solve for x and choose the larger root because of the physical sense from Fig. B.

$$\Rightarrow x = q\sqrt{1 - \left(\frac{P}{2R}\right)^2} + \sqrt{r^2 - \left(\frac{qP}{2R}\right)^2}$$

From Fig. B, we get $h = R - x$,

$$h = R - (R - r)\sqrt{1 - \left(\frac{P}{2R}\right)^2} - \sqrt{r^2 - \left(\frac{(R - r)P}{2R}\right)^2} \quad (10)$$

APPENDIX C

From Fig. 13 and using Pythagora's theorem, we get

$$R^2 = (R - \delta_{cc})^2 + \frac{l^2}{4}$$

$$\Rightarrow 2R\delta_{cc} - \delta_{cc}^2 = \frac{l^2}{4}$$

Since δ_{cc} is usually very small, we get

$$\delta_{cc} = \frac{l^2}{8R} \quad (22)$$

From Eq. (22),

$$\delta_{cc} = \frac{l_{cc}^2}{8R_{cc}} \quad \text{and} \quad \delta_{cl} = \frac{l_{cl}^2}{8R_{cl}}$$

$$\Rightarrow \frac{\delta_{cl}}{\delta_{cc}} = \left(\frac{l_{cl}^2}{l_{cc}^2}\right)\left(\frac{R_{cc}}{R_{cl}}\right)$$

From Fig. C, we get

$$\frac{l_{cl}}{l_{cc}} = \frac{R_{cl}}{R_{cc}}$$

$$\Rightarrow \delta_{cl} = \left(\frac{R_{cl}}{R_{cc}}\right)\delta_{cc} = \left(\frac{R + r}{R}\right)\delta_{cc} \quad (23)$$

Likewise, Eq. (24) can be obtained.

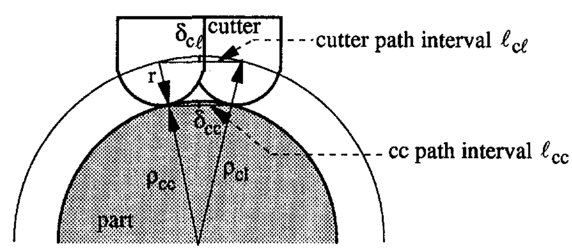

Fig. C Illustration of derivation of Eq. (23). The cutting path direction is into/out of the paper.