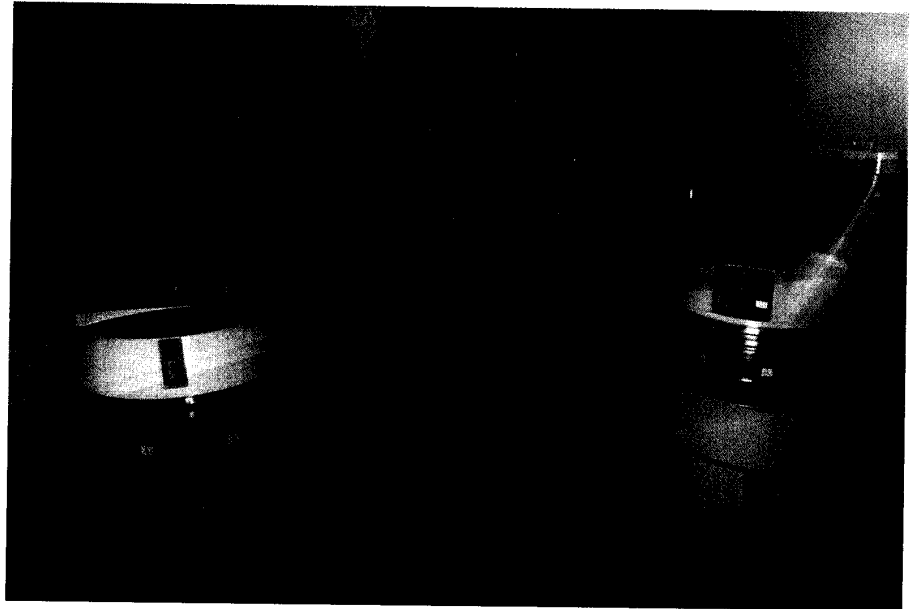


# Cross-Coupling Motion Controller for Mobile Robots

L. Feng, Y. Koren, and J. Borenstein

The design and implementation of a cross-coupling motion controller for a differential-drive mobile robot is described here. A new concept, *the most significant error*, is introduced as the control design objective. Cross-coupling control directly minimizes *the most significant error* by coordinating the motion of the two drive wheels. The cross-coupling controller has excellent disturbance rejection and therefore is advantageous when the robot is not loaded symmetrically or has large friction in its drive mechanism and especially when it is instructed to follow curved paths. The cross-coupling controller is analyzed and experiments are conducted to evaluate its performance. The experimental results show that cross-coupling control yields substantially smaller position and orientation errors than conventional methods.



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## Improving Trajectory Tracking Accuracy

Mobile robots are utilized in a variety of applications, including defense, nuclear power plant maintenance, waste management, assistance to the disabled, material handling, security, and household service [1]. A primary limitation of existing mobile robots is their accuracy in trajectory tracking. One way to address this problem is by improving the accuracy of the low-level motion controller [2]. The proposed motion controller is comprised of three main components. The first is the trajectory interpolator, which utilizes the inverse kinematics to generate the desired velocity profiles for each wheel from a given desired path. The second component is the wheel-level controller, which

controls the motion of the drive-wheels. The third component is the vehicle-level controller which uses a cross-coupling (CC) controller for coordination of both drive loops [3]. Here we deal with a low-level controller comprised of the vehicle-level and wheel-level controllers.

A term that is used throughout this paper is *the most significant error*, which is defined as the error that has the largest impact on motion accuracy. For example, in machine tool control, *the most significant error* is the contour error [4]. In mobile robot control, *the most significant error* is the orientation error. Note that the individual error in each loop does not give a full description of the real robot motion error. In order to accurately control the robot motion, a motion controller that has the following two characteristics is needed.

1. Direct control of the most significant error, rather than the error in each individual drive loop.
2. Direct coordination of the velocities of both drive wheels.

We would like to elaborate on the last point. The accuracy of the robot motion depends on how well the wheel velocities are

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coordinated. In the case of a differential-drive robot, there are two drive wheels driven by two different motors that are controlled independently. The steering is accomplished by the difference in speed of the two drive wheels. In conventional mobile robot controllers, each drive loop receives no information about the other, and any disturbance in one loop causes an error that is corrected only by this loop, while the other loop carries on as before. This lack of coordination causes an error in the resultant path [5]. Cross-coupling control is used to remedy this problem by sharing the feedback information of both control loops.

We will discuss the basic idea of cross-coupling and show how it can be used to improve the motion control performance. In the following section, we analyze the sources of trajectory errors in mobile robot. In the third section, we give the mathematical analysis of the CC-controller, and next, the design of the motion controller. After that, we provide simulation and experimental results, and our conclusions.

### Motion Error Analysis

The velocity kinematics of a differential drive mobile robot are given by [2]

$$\dot{x} = \frac{v^L + v^R}{2} \sin \theta \quad (1)$$

$$\dot{y} = \frac{v^L + v^R}{2} \cos \theta \quad (2)$$

$$\dot{\theta} = \frac{v^R - v^L}{b_w} \quad (3)$$

where  $x$ ,  $y$  are the position of the robot in the world coordinate system (mm),  $\theta$  is the orientation,  $\dot{x}$ ,  $\dot{y}$  are robot velocities (mm/s),  $\dot{\theta}$  is the angular velocity of the robot (1/s),  $v^L$ ,  $v^R$  are the linear velocities output of the left and right wheels (mm/s), and  $b_w$  is the distance between the two drive wheels (wheel base) (mm).

We can largely classify the error sources into two categories: internal errors and external errors. The internal errors are the errors that can be detected by the wheel motion information. The external errors are the errors that only become apparent when the robot wheels interact with the environment; that is, external errors can only be detected by absolute robot motion measurements. In this study, we deal with the control of internal errors.

The main sources of internal errors are:

1. *Different drive loop parameters.* For a differential-drive robot, when the two drive loops have different parameters (e.g., time constants and loop gains), the responses of the two loops will be different, and the result is an error in the path.

2. *Different disturbances acting on individual drives.* One example is different bearing frictions [5]. The difference in disturbances affects the transient response and the steady-state response.

3. *Inability to track nonlinear trajectories.* In tracking a general nonlinear trajectory, the reference inputs to the drive loops are also nonlinear. A conventional control system has lag errors in tracking nonlinear inputs.

The motion error of the robot can be decomposed into the components shown in Fig. 1:  $e_\theta$ ,  $e_c$ , and  $e_t$ . The first component is the orientation error  $e_\theta$ , which is defined as the difference between the real robot orientation and the desired robot orientation. It is *the most significant error* as far as motion accuracy is concerned, because the orientation error will result in a contour error, which grows with the distance traveled.

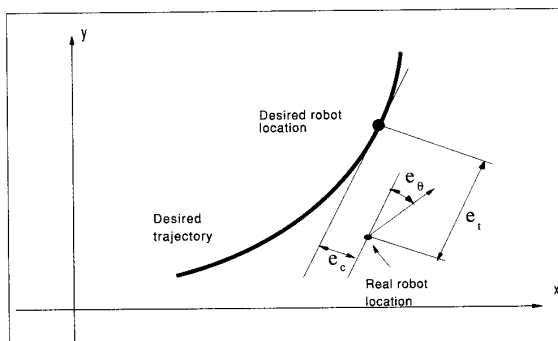


Fig. 1. Motion error decomposition.

The contour error  $e_c$  is defined as the distance between the actual robot position and the desired robot position in the direction perpendicular to the direction of travel. The third error is the tracking error  $e_t$ , which is the distance between the actual position and the desired position in the direction of travel.

The tracking error does not have a significant effect on the motion accuracy of the robot, and can be controlled by adjusting the robot traveling speed as desired. The contour error is the direct result of the orientation error. However, we cannot control both errors at the same time with a differential-drive robot. Among these three errors, the orientation error has the largest impact on the motion accuracy since it results in accumulation of contour and tracking errors. Elimination of the orientation error would cause the robot to follow a path that is parallel to the desired path, and consequently the contour error would be bounded.

### Cross-Coupling Control

The task of the vehicle-level controller is to coordinate the motion of the drive loops. In the case of a differential-drive robot, most conventional controllers consist of two individual control loops, one for each motor. Motion coordination is achieved by adjusting the reference velocities of the control loops, but one drive loop receives no information regarding the other. The error in each loop is treated as the primary error, although it normally does not present the most significant motion error. With this configuration, any load disturbance in one drive loop causes an error that is corrected only by its own loop while the other loop carries on as before, and, consequently, an orientation error is caused in the resultant path. An improvement in the path accuracy can be achieved by providing cross-coupling control, whereby *the most significant error* can be controlled directly.

The cross-coupling concept was first introduced by Koren for machine tool control [6]. In machine tool servo control, the main idea of cross-coupling control is based on calculation of the actual contour error, multiplying it by a controller gain, and

feeding the result back to the individual loops. Kulkarni and Srinivasan [7] has presented cross-coupling compensator structures for two-axis machines. Both simulation and experiments demonstrated improved contouring accuracy in machining straight-line and circular contours. Kulkarni [8] also presented an optimal cross-coupling control structure which result in zero steady-state contour error in linear cuts and small steady-state contour error in circular cuts. More recently, a variable-gain CC-controller for milling machines, which gives excellent contour tracking for nonlinear contours was proposed [4]. The experimental results show significant improvements (up to one order of magnitude) over conventional controllers; especially at high milling velocities.

In mobile robot control, Samson studied the feedback control and stabilization of a two-wheel-driven nonholonomic cart [9]. De Wit studied the control of a two-degree-of-freedom robot under path and input torque constraints [10]. Yamamoto studied the coordination between a manipulator and the locomotion of its mobile base. However, cross-coupling control has not been widely studied in the field of robotics. Most work is done in the area of machine tool control where the objectives, requirements, and operating conditions are significantly different from mobile robot applications. Fujii studied a cross-coupling motion control system in which each loop used the position error of the other loop [12]. A CC-controller was implemented by Borenstein and Koren on a mobile robot and demonstrated significant reduction in the position errors by experiments [5]. The system is proved to guarantee a zero steady-state orientation error despite continuous torque disturbances. In a broad sense, cross-coupling control includes all control schemes that use feedback information from more than one control loop to control a composite error (normally calculated from individual loop errors) rather than each individual loop error. Strictly speaking, any mobile robot motion controller with vehicle or path-level corrections are cross-coupling controllers although they are not explicitly identified as such. Nelson developed a mobile robot motion controller for a tricycle

type robot [13]. The controller has a path-level controller that aims at eliminating both tangential and normal errors. Dead-reckoning information is used in error calculation. Hongo developed a motion controller aimed at controlling a composite error in the form of  $e = k_0e_0 + k_1e_1 + k_2e_2$ , where  $e_0$  is velocity error,  $e_1$  is position error,  $e_2$  is directional error, and  $k_0, k_1, k_2$  are weighting factors. In this article, we concentrate on low-level control with only wheel encoder feedback.

Most of the work in cross-coupling control concentrates on machine tool control. However, the difference in the kinematics of mobile robots and machine tools has important consequences: in two-axis machine tool control, the two axes are perpendicular to each other and their motions are completely decoupled. It is easy to find the relations between the machined part errors and the drive axis errors. But in the case of a differential-drive mobile robot, the two axes are parallel to each other, and their motions are coupled through the robot body. The relations between the motion errors and the drive axes errors are not well known except for the orientation error.

Furthermore, since the axes in machine tools are perpendicular, the tool can be moved to any direction to compensate for the path errors. By contrast, a differential-drive robot can not move in the direction that is perpendicular to the general direction of motion. Another difference lies in the fact that in machine tool control, the contour error is the most critical error, while in the case of mobile robots, the most significant error is the orientation error.

In straight line motion, at each sampling period, the change in the orientation error  $\Delta e_\theta$  is

$$\Delta e_\theta = \frac{\Delta q^R - \Delta q^L}{b_w}$$

where  $\Delta q^L$  and  $\Delta q^R$  are the left and right wheel displacements during the sampling period. The total orientation error  $e_\theta$  is

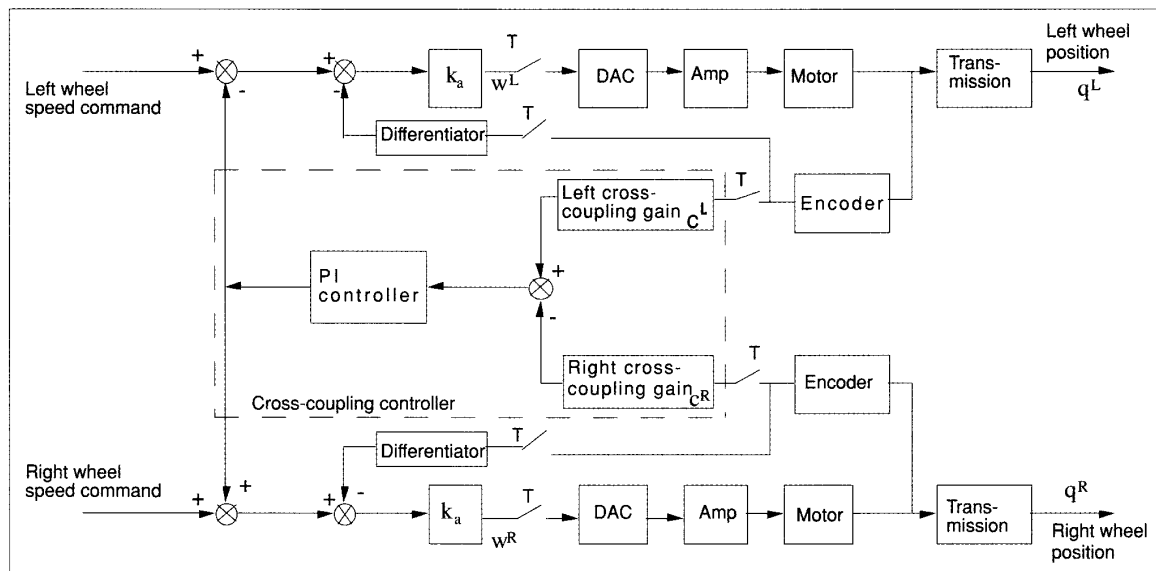


Fig. 2. The block diagram of a cross-coupling controller.

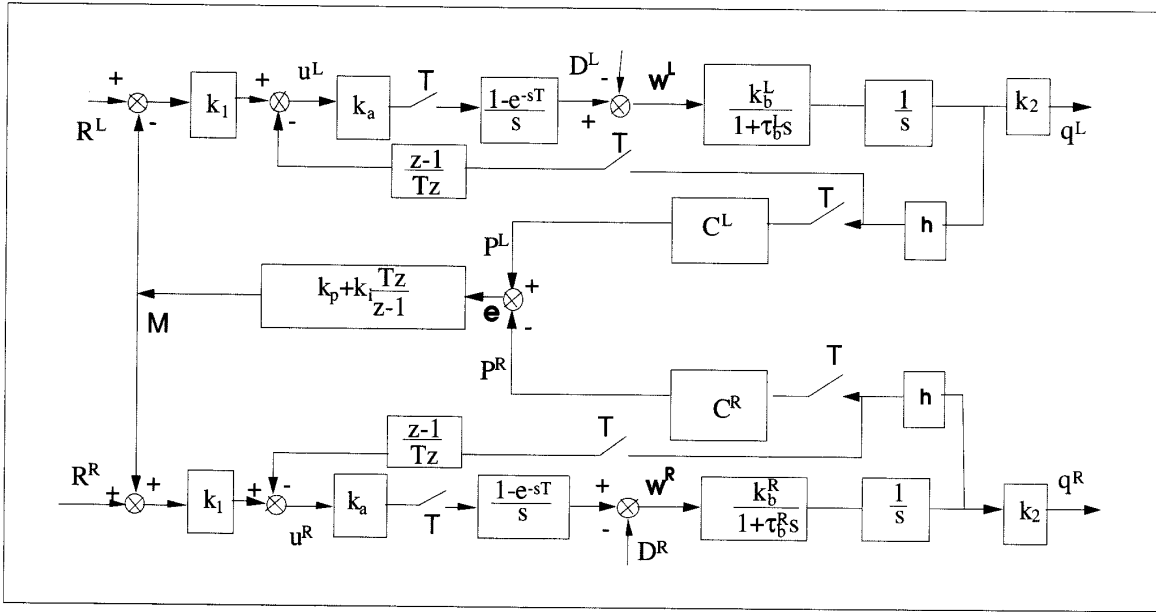


Fig. 3. Major components of the cross-coupling controller.

$$e_{\theta} = \sum \Delta e_{\theta} = \frac{q^R - q^L}{b_w} \quad (4)$$

where  $q^L$  and  $q^R$  are the total left and right wheel displacements.

The proposed CC-controller is shown in Fig. 2. In order to allow the robot to move along curved paths, we introduce the cross-coupling gains  $c^L$  and  $c^R$ . The path of a mobile robot is often composed of linear and circular segments. In order to define a circular path, the center of the robot must move along a circle of radius  $R$ , and the speed of the two drive wheels should be

$$v^L = \omega(R - b_w/2) = v(1 - b_w/2R) \quad (5)$$

$$v^R = \omega(R + b_w/2) = v(1 + b_w/2R) \quad (6)$$

where  $\omega$  is the angular velocity of the robot along the circular path and  $v$  is robot velocity. We can observe that

$$\frac{v^R}{v^L} = \frac{1 + b_w/2R}{1 - b_w/2R}$$

The circular motion can be realized by supplying the references in (5) and (6) and setting

$$c^L = 1 \quad (7)$$

$$c^R = \frac{v^L}{v^R} = \frac{1 - b_w/2R}{1 + b_w/2R} \quad (8)$$

The linear motion is a particular case in which  $R = \infty$ . In order to track a general nonlinear trajectory, its radius of curvature is calculated at each sampling period, and substituted into (8), which results in a CC-controller with time-varying gains.

It is worthwhile to mention that the cross-coupling gains  $c^L$  and  $c^R$  may be further modified to compensate for *known* external errors. For example, the diameters of the drive wheels usually differ slightly (e.g., due to manufacturing tolerances). If, for example, the left drive wheel diameter  $d^L$  is larger than the right drive wheel diameter  $d^R$  and we give the same speed commands to both control loops, the result would be a circular path. However, we can multiply  $c^L$  and  $c^R$  with the correction factor  $c_{1L} = 1$  and  $c_{1R} = d^L/d^R$ , thereby effectively compensate for this error.

Nevertheless, since there are many factors affecting the path errors changing with the operation of the robot and the environment, a fixed set of encoder compensation gains cannot provide satisfactory performance over a wide range of operating conditions. An improvement in robot accuracy is achieved when the gain values are adjusted adaptively to compensate for motion errors. This process of adaptive compensation is addressed in a related study [12], but these correction factors are not used here.

When CC-controllers are used, the orientation error in (4) becomes

$$e_{\theta} = \sum \Delta e_{\theta} = \frac{c^R q^R - c^L q^L}{b_w} \quad (9)$$

In the proposed cross-coupling architecture, the model of the orientation error is built in real time as shown in Fig. 2. This signal is used to generate compensating signals for the two axes. In general, the selection of the *most significant error* is not unique and depends on several factors.

1. *The application.* An application determines the requirements on the robot motion. If the robot needs to move in confined

space, the contour and orientation errors are very important. But in other applications where the absolute speed is important, the tracking error is more important.

2. *Kinematics.* The kinematic structure determines the motion of the robot and how the error correction can be implemented.

3. *Available information.* The most significant error needs to be calculated in real time from the information available from the system.

### Controller Design

In this section, we give the mathematical description of the CC-controller and discuss its properties.

Since the orientation error is used as the most significant error in this study, the proposed cross-coupling control scheme controls the orientation error of the robot. Fig. 3 shows the mathematical representation of the various components in the controller. From Fig. 3, we obtain

$$q^L(s) = \frac{k_a k_b^L (1 - e^{-sT_i}) u^L(s)}{s^2 (1 + s\tau_b^L)} - \frac{k_b^L D^L(s)}{s(1 + s\tau_b^L)} \quad (10)$$

where  $q^L, q^R$  are the accumulated left and right wheel displacements since the beginning of motion (mm),  $u^L, u^R$  are the inputs to the left and right motor controllers,  $k_a$  is the proportional gain of the inner velocity loop,  $D^L, D^R$  are the disturbance in the left and right drive loops,  $\tau_b^L, \tau_b^R$  are the time constants of the left and right drive loops (s), and  $k_b^L, k_b^R$  are the left and right motor gains ((rev/s)/V).

By applying the z-transform on this equation, we get

$$q^L(z) = \frac{k_a k_b^L (n_1^L z + n_0^L)}{(z-1)(z-r^L)} u^L(z) - \frac{k_b^L (1-r^L)z}{(z-1)(z-r^L)} D^L(z) \quad (11)$$

where

$$\begin{aligned} r^L &= e^{-T/\tau_b^L} \\ r^R &= e^{-T/\tau_b^R} \\ n_0^L &= \tau_b^L - \tau_b^L r^L - T r^L \\ n_1^L &= T - \tau_b^L + \tau_b^L r^L \\ n_0^R &= \tau_b^R - \tau_b^R r^R - T r^R \\ n_1^R &= T - \tau_b^R + \tau_b^R r^R \end{aligned}$$

Similar equations can be obtained for the right drive loop. From Fig. 3, we also have

$$e(z) = c^L q^L(z) - c^R q^R(z) \quad (12)$$

$$m(z) = (k_p + k_i \frac{zT}{z-1}) e(z) \quad (13)$$

$$u^L(z) = (R^L(z) - m(z)) k_1 - h \frac{q^L(z) \frac{z-1}{Tz}}{k_2} \quad (14)$$

$$u^R(z) = (R^R(z) + m(z)) k_1 - h \frac{q^R(z) \frac{z-1}{Tz}}{k_2} \quad (15)$$

where  $R^L, R^R$  are the reference inputs to the left and right drive loops (mm/s),  $c^L, c^R$  are the encoder compensation gains for the left and right wheel control loops (can be adjusted to compensate for the path errors),  $e$  is the position difference of the two loops (mm),  $m$  is the correction signal generated by the CC-controller,  $k_p, k_i$  are the proportional and integral gains of the cross-coupling loop,  $k_1, k_2$  are conversion factors, and  $h$  is the feedback gain =  $4 \times \text{encoder resolution} \times \text{sampling period}$ .

The error  $e$  in (12) is proportional to the robot orientation error  $e_\theta$ , i.e.,  $e = e_\theta b_w$ . If we neglect the disturbances, the inputs to the motors  $w^L$  and  $w^R$  (Fig. 3) can be written as

$$w^L = k_a (k_1 R^L - v^L) - \frac{k_a k_1}{b_w} (k_p e_\theta + k_i \int e_\theta dt) \quad (16)$$

$$w^R = k_a (k_1 R^R - v^R) + \frac{k_a k_1}{b_w} (k_p e_\theta + k_i \int e_\theta dt) \quad (17)$$

The first term in (16) and (17) represents the conventional proportional (P) control, which controls the general motion of the robot. The second term represents the cross-coupling control, which generates compensating signals when  $e_\theta \neq 0$  to turn the robot to the correct orientation. For example, when the robot follows a straight line path and there is no orientation error, i.e.,  $e_\theta = 0$ , we have only the conventional P-control. However, when  $e_\theta \neq 0$ , say  $e_\theta > 0$  a compensating signal will be generated by the CC-controller to slow down the left wheel and speed up the right wheel until the robot has the correct orientation. From this process, we can observe that the CC-controller directly controls the most significant error, in this case, the orientation error. The error correction happens in both drive loops simultaneously and the motion of the two axes is coordinated.

To simplify the steady-state analysis, we do not consider the disturbances and assume that the speed inputs are step functions, i.e.,

$$R^L(z) = \frac{R^L z}{z-1}$$

$$R^R(z) = \frac{R^R z}{z-1}$$

Then at steady state, we have,

$$v^L = \frac{hk_a k_b^L k_b^R c^R (R^L + R^R)}{hk_a k_b^L k_b^R (c^L + c^R) + k_b^L c^L + k_b^R c^R} \quad (18)$$

$$v^R = \frac{hk_a k_b^L k_b^R c^L (R^L + R^R)}{hk_a k_b^L k_b^R (c^L + c^R) + k_b^L c^L + k_b^R c^R} \quad (19)$$

The velocity ratio of the two drive wheels is inversely proportional to the ratio of the two encoder compensation gains  $c^L/c^R = v^R/v^L$ .

If the desired steady-state velocities are  $v_0^L$  and  $v_0^R$ , we let

$$c^L = \alpha v_0^R \quad (20)$$

$$c^R = \alpha v_0^L \quad (21)$$

Substituting (20) and (21) into (18) while setting (18) equal to  $v_0^L$ , and after rearranging, we get

$$R^L + R^R = \frac{1 + hk_b^R k_a v_0^L}{hk_b^R k_a} + \frac{1 + hk_b^L k_a v_0^R}{hk_b^L k_a} \quad (22)$$

If we choose  $R^L$  and  $R^R$  in the following manner, we obtain the steady-state velocity equal to their desired values.

$$R^L = \frac{1 + hk_b^R k_a v_0^L}{hk_b^R k_a} \quad (23)$$

$$R^R = \frac{1 + hk_b^L k_a v_0^R}{hk_b^L k_a} \quad (24)$$

Thus we can select the appropriate encoder compensation gain ratio to achieve the desired wheel speed ratio.

Now we will discuss the effect of the disturbances. Let us assume that the two motors have identical parameters, i.e.,  $k_b L = k_b R = k_b^0$  and  $\tau_b L = \tau_b R = \tau_b^0$ , but there are step disturbances in both loops, i.e.,

$$D^L(z) = \frac{D^L z}{z-1}$$

$$D^R(z) = \frac{D^R z}{z-1}$$

Then, by applying the final value theorem, we can find the steady-state velocities of the two drive wheels,

$$v^L = \frac{c^R}{c^L + c^R} \frac{k_b}{1 + hk_b k_a} [hk_a (R^L + R^R) - \frac{k_2}{(1-r)\tau_b} (D^L + D^R)] \quad (25)$$

$$v^R = \frac{c^L}{c^L + c^R} \frac{k_b}{1 + hk_b k_a} [hk_a (R^L + R^R) - \frac{k_2}{(1-r)\tau_b} (D^L + D^R)] \quad (26)$$

As we can see from the above equations, at steady state, we still have  $v^L/v^R = c^R/c^L$ . That is, even though there are different constant disturbances acting on the control loops, the orientation of the robot is not affected by the disturbances. This is one of the most important properties of the CC-controller. It is capable of dealing with disturbances and variations of parameters in the forward loop, e.g., variations in motor parameters, differences in bearing friction, and unsymmetric load.

Based on the above analysis, we conclude that:

1. At steady state,  $v^L/v^R = c^R/c^L$ .
2. The steady-state orientation error caused by the continuous disturbances is zero.

This controller is PI control on orientation and P control on velocity. A compromise is made between driving at the desired speed and assuming the desired orientation. Since the orientation is controlled by a PI controller, it is guaranteed for zero steady-state orientation error. The wheel speeds are directly measurable. The method presented is one way to implement cross-coupling control without using high-level or external sensory information.

## Experimental Results

We tested the performance of our cross-coupling control method on the commercially available *LabMate* platform (Fig. 4). The *LabMate* has a square shape of 75 cm by 75 cm, and has a maximum speed of 1 m/s.

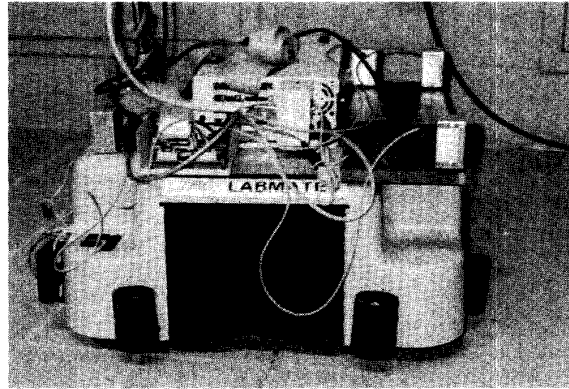


Fig. 4. Experimental robot.

The dc motors are connected to the drive wheels through transmissions. The governing dynamic equation for the motor-wheel system [15] is

$$\tau \frac{d\omega}{dt} + \omega = K_b V - K_b R^T / K_t \quad (27)$$

where  $\tau$  is the mechanical time constant of the system (s),  $T_s$  is the disturbance torque from the wheel friction, bearing friction, etc. ( $\text{kg}\cdot\text{m}^2$ ),  $K_t$  is the torque constant of the motor (Nm/A),  $K_b$  is the gain of the motor ((rev/s)/V), and  $V$  is the input voltage (V).

The parameters of the drive loops were estimated through open loop experiments. On our platform, the parameters of the left motor are  $\tau_b L = 165$  ms and  $k_b L = 0.225$  (rev/s)/V, whereas the parameters of the right motor are  $\tau_b R = 166$  ms and  $k_b R = 0.237$  (rev/s)/V.

In designing the CC-controller, there are four parameters that need to be adjusted. The first two are the encoder compensation gains  $c^L$  and  $c^R$ . Since this paper focuses on internal errors, we neglect external errors and select  $c^L/c^R = R^R/R^L$ . The proportional and integral gains of the CC-controller are selected through computer simulation and then fine-tuned through experiments.

A series of experiments were conducted on a *LabMate* mobile robot to evaluate the performance of the new controller. The system controller can be easily switched from a P-controller to the CC-controller. The gain of the proportional feedback loop is  $k_a = 2$ , the cross-coupling gains are  $k_p = 5.13$  and  $k_i = 10.22$ , the sampling periods for both loops are 4 ms, and the slope of the input (i.e., the acceleration) is  $1000 \text{ mm/s}^2$ , and the reference velocity for both wheels is  $200 \text{ mm/s}$ . Fig. 5(a) shows the difference in the distance traveled (i.e., the encoder reading) by the two wheels using the two controllers. We can see that when only proportional control is used, the difference keeps increasing because the two motors have different parameters and, in turn, the orientation error increases and the robot diverges from the desired path. By contrast, under the cross-coupling control, when there is a difference, a correction signal is generated to reduce the error, and the robot travels in the direction of the desired path. The correction signal is shown in Fig. 5(b). We can observe from Fig. 5(b) that the average correction signal is not zero, but has a negative value. This fact also suggests that the two drive loops have different parameters.

A path following experiment is also conducted to test the controller. The robot is instructed to follow a  $2 \text{ m} \times 2 \text{ m}$  square using cross-coupling and P-controllers. The objective of this experiment is to compare the repeatability rather than absolute accuracy. To eliminate slippage, the robot is instructed to move at a slow speed of  $200 \text{ mm/s}$ , and it stops at the corner to turn on the spot.

In order to eliminate the effects of systematic error, the experiments are conducted in both clockwise and counterclockwise directions; the results are shown in Fig. 6(a) and (b), respectively. To reduce the effects of random errors, each experiment was repeated 10 times. We can observe that the CC-controller gives a much better repeatability. The results are consistent in both clockwise and counterclockwise motion. The robot does not return exactly to the start position because there are some systematic errors in the robot. However, the systematic error can be effectively compensated as long as the system has good repeatability.

The improvement in accuracy is obtained because of the cross-coupling controller can effectively reject internal distur-

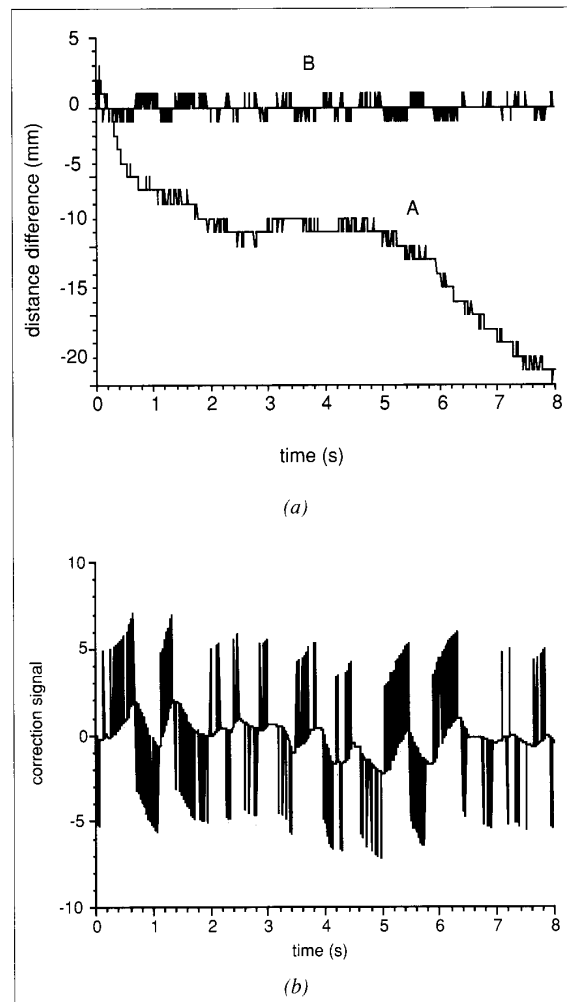


Fig. 5. (a) Difference of the distance traveled by the two drive wheels using proportional and cross-coupling controllers. Trace (A) of the graph is using proportional controller, (B) is using cross-coupling controller. (b) Corresponding correction signal generated by the cross-coupling controller.

bances (which are dependent on the external factors, such as the orientation of caster wheels and the floor frictions).

Cross-coupling control has been used for nonlinear tracking in machine tool control and the results are very encouraging [4]. However, in the case of mobile robot control, nonlinear tracking has not been widely studied because of the low accuracy requirements. In the literature, most common test trajectories are line segments, squares and circles. The proposed controller works best with long straight lines or circular segments. Since it makes a compromise between the desired velocity and desired orientation, this will cause errors in nonlinear trajectory following. The problem of nonlinear tracking needs to be further studied.

The above comparison was done with a P-controller. When proportional-integral (PI) controllers are used to control the system, there will be no steady-state error, i.e., the orientation

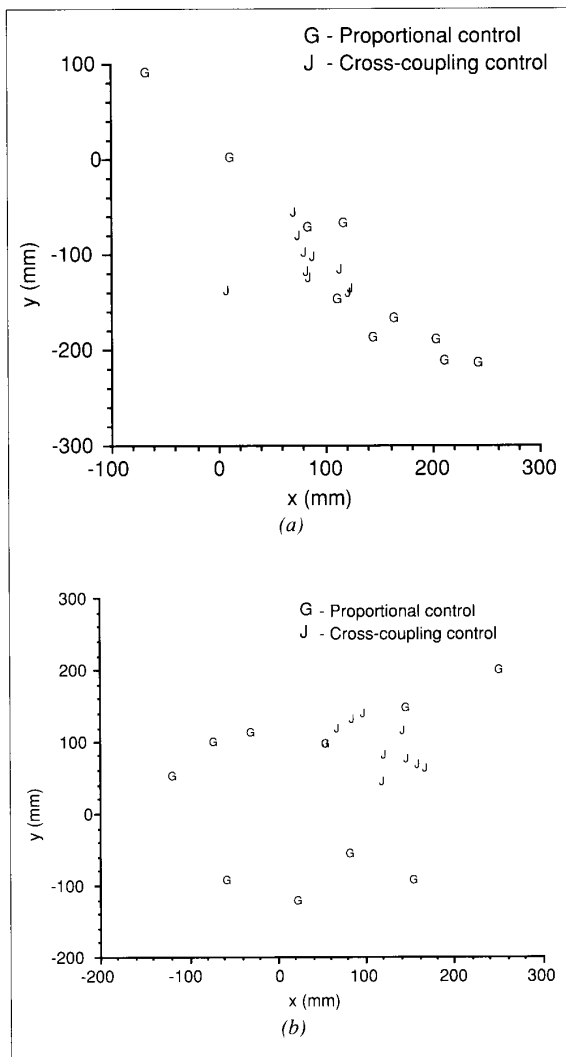


Fig. 6. Comparison of the motion accuracy of the cross-coupling and the proportional controllers: (a) following a  $2\text{ m} \times 2\text{ m}$  square path (clockwise), and (b) following a  $2\text{ m} \times 2\text{ m}$  square path (counterclockwise).

error will not keep increasing with the distance traveled. However, the problem with utilizing PI-controller is that the mobile robot overshoots the target position. The overshoot can be reduced only by decelerating the robot a substantial distance before stopping, which, in turn slows down the robot. In addition, the CC-controller offers several advantages over a conventional PI-controller.

The CC-controller rejects disturbances better, and has a shorter settling time. Computer simulation is conducted to verify this point. In the simulation, the same amount of random disturbance is added in both PI and cross-coupling control loops.

The responses of the two controllers are compared. Fig. 7(a) shows the difference in distance traveled by the two drive wheels. As we can observe from the result, the CC-controller is much more effective in controlling the error growth. The resultant orientation

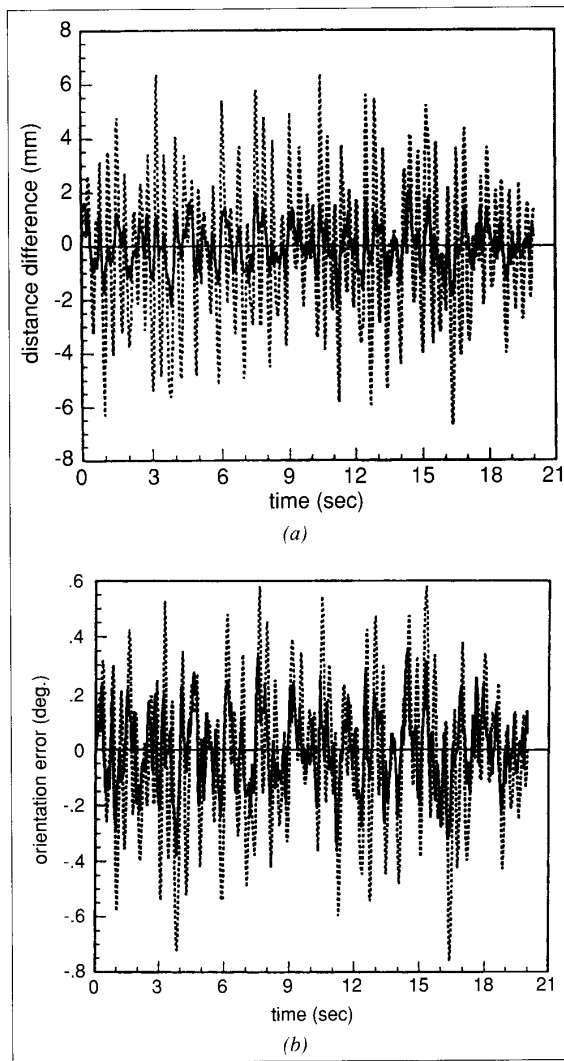


Fig. 7. Simulation comparison of PI and cross-coupling controllers: (a) difference of the distance traveled by the two wheels, and (b) orientation error. Solid lines indicate cross-coupling control; dotted lines, PI control.

error is shown in Fig. 7(b). The amplitude of variations in the angle is two to three times smaller with cross-coupling.

The proposed method is to be used as low-level motion control, and requires only wheel encoder feedback. Its major task is to realize the command generated by a higher-level controller, which will incorporate additional sensory information, such as range and directional information provided by ultrasonic sensors. The low-level controller is designed to be independent of external sensors so that it can be effective even when there is no external sensory information available.

### Controlling the Most Significant Error

We have discussed and evaluated a CC-motion controller for a differential-drive mobile robot to minimize orientation error.



The CC-controller coordinates the motions of the two drive loops. This controller is intended as a low-level controller to be used in combination with high-level controllers. The experimental results showed that it is very effective in compensating for internal errors, such as differences in motor parameters and different bearing frictions. The most important advantage of cross-coupling control is that it directly controls the most significant error (which is the orientation error in mobile robots), while conventional controllers minimize the errors in each drive loop. The other advantage of the cross-coupling control is that the corrections occur in both loops simultaneously. As a result, it has short settling time as well as excellent disturbance rejection capability. In addition, a pair of encoder compensation gains were introduced to allow compensation for the external errors through calibration. Note, however, that absolute measurements of the robot position are needed for calibration [17]. Furthermore, when multiple mobile robots are needed to perform cooperatively a single task, e.g., carrying very long objects, the coordination among the robots is critical, and higher-level cross-coupling control can be a valuable tool to solve the problem [18].

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