Optimal control of reassembly with variable quality returns in a product remanufacturing system

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ABSTRACT

We consider the assembly system of modular products in the remanufacturing environment subject to used module returns with uncertainties in terms of timing, quantity and quality. n customer demand classes of remanufactured products are classified by $l = 1,\ldots,n$ levels of product quality. Returned modules arrive according to a compound Poisson process and each demand class follows an independent Poisson process. Returned modules can be stocked for reassembly upon demand requests, but incur a holding cost. We formulate the problem as a Markov decision process and show that the reassembly and inventory control policy is a state-dependent threshold type policy.

1. Introduction

This paper concerns optimal policy for modular product reassembly within a remanufacturing setting where a firm, receiving end-of-life product returns with quality variance, faces multiple classes of demands for remanufactured products.

Remanufacturing has the ability to repair degraded components and put the product into a second life cycle, thus retaining the value of the extracted and refined material, as well as some fraction of the original manufactured value. Remanufacturing includes disassembling, cleaning, refurbishing, and reassembling parts to produce a "like new" product [1,2]. For automotive and heavy equipment, engine and fuel system components are widely remanufactured. In today's market, many products are remanufactured and reused, oftentimes without customer knowledge [3].

The major difference between remanufacturing and manufacturing systems lies in the supply side. In a remanufacturing system, supply is largely exogenous, and the timing, quantity, and quality of supply are much more uncertain than those in conventional manufacturing systems. Such uncertainties could result in several challenging issues in re-assembly [4]. First, components and modules within a single used product can typically vary in quality; with some modules have higher quality level and longer residual life than others. Second, customer demands are not necessarily differentiated by product type but rather by product quality. Quality-based demand classes may have different costs and benefits so that it is more important to satisfy one class than the others [5]. Some inventory control models for assemble-to-order (ATO) manufacturing systems deal with multiple demand classes where many different types of customized products are produced using some common components or modules and share the component inventories. In this paper we will focus on multiple classes of customer demand where this is not the case, i.e., demand for a single remanufactured product can be classified into different quality grades which are determined by the quality of each module. Reassembly policy and inventory control in such situations are more complicated than traditional assembly systems because the firm must jointly manage inventories across variable quality modules as well as different types of module. Evidently, new models and methods are demanded to address these challenging and important issues in remanufacturing system.

2. Literature review

Although there is a growing body of literature addressing production planning and control for remanufacturing [6], most of them assume a single quality grade for all returns. We found very few that considers different quality of returns with uncertainty issue. Souza et al. modelled the remanufacturing system as a multi class open queuing network where they classify returned product according to quality levels and dispatch them to different remanufacturing stations based on dispatching rules [7]. Ferguson et al. (2006) investigated a remanufacturing planning problem when returned products have different quality levels and demand has a network-like structure [8]. They formulated the problem as a linear program and provided optimal decisions on the quantity to remanufacture or to salvage for each quality level in each period. Extended study were found in "multi-period remanufacturing planning with uncertain quality of inputs" where a stochastic program was formulated where the quality levels of returned cores may take on any configuration of bad to good quality levels based on pre-established probabilities [9]. All these models considered the variant quality levels at the single product level, however, ignored the module-level quality variation because the internal module-to-product assembly hierarchy was not taken into account for decision-making. In fact, for "cradle-to-grave" product design, structuring and modularisation of products are exceptionally important for disassembly and reassembly in remanufacturing

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because they can facilitate customization and repeated utilization of single modules from different quality classes [10,11].

No research literature has explicitly considered the problem of reassembly planning and inventory control of a multi-module product with uncertain quality. In this paper, we aim to provide insights to the abovementioned problems by considering a reassemble-to-order setting in the remanufacturing environment. We assume that used product returns are an exogenous process, which refers to the assumption that return quantity, quality, and timing are all out of the direct control of the remanufacturer. Under this classic assumption of exogenous and uncontrollable return flows, it is believed that remanufacturing operations are more complex to plan and control than traditional manufacturing operations [12]. The primary reason for this complexity is the high degree of variability in the quality of the used products that serve as input materials for the assembly process.

Our model is mostly related to the literature of inventory rationing for assemble-to-order (ATO) system in conventional manufacturing. In conventional manufacturing systems, short product life cycle and requirement for quick response to market changes have pushed ATO systems more advantageous to the traditional "assemble-to-stock" systems, because ATO keep inventory only for modules and not assemble the end products until demand is realized, have become prevalent because the potential of improving customer service while at the same time reducing expensive finished-goods inventories [13,14].

3. Problem description

We consider a reassemble-to-order system consisting of a single remanufactured product assembled from m modules and each type of returned module has n different quality grades. There are accordingly n demand classes categorized by the product quality grades because customers' preference for the remaining useful life (quality) of the secondary life products may differ by future usage and price. We assume that modules in a product are not interchangeable and a reassembly process requires one unit at a time on m modules. Unit inventory costs may vary from module to module and also vary with the quality of the modules.

We initially consider a two-module two-quality grade product problem ($m = 2$, $n = 2$). We assume that the "weakest link property" applies to the quality of an assembly structure, saying that a higher quality class of demand can only be satisfied by assembly of two higher quality modules or new product while a lower quality class of demand can be satisfied by lower quality modules if they are all available in stock, otherwise make decision on whether to use a higher quality module to substitute if one of the lower quality modules is missing.

The remanufacturing system consists of two inventories where the quality-specific modules are stocked before demand is realized (See Fig. 1). It is assumed that inter-arrival times of returns of module k, ($k = 1, 2$) are independent and exponentially distributed with mean μ_k^{-1} and the probability that the module has quality l is p_l ($l = 1, 2$). Inventory costs are incurred at a constant rate of h_k^l for each unit of returned module k with quality l kept in stock. The reassembly action is triggered by an order request from one of the two classes of demand ($l = 1, 2$). During reassembly, a quality-specific module is turned into a final remanufactured product through operations that include joining of modules with different quality grades shared with both demand classes.

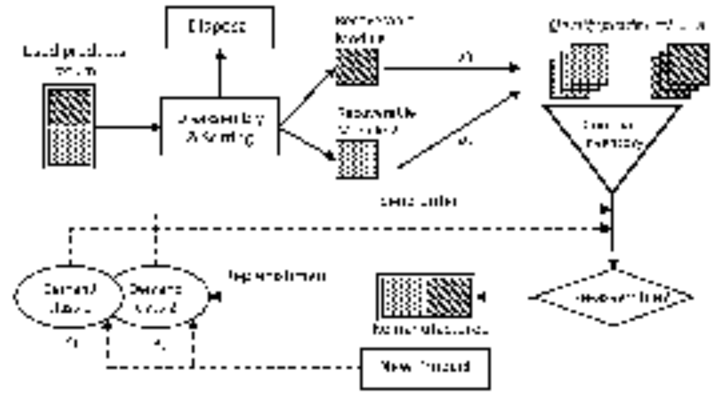

Fig. 1. Reassemble-to-order system in remanufacturing environment.

Demand for the remanufactured product from each class l, takes place continuously over time according to an independent Poisson process with rate λ_l. Demand of a higher quality level ($l = 2$) can be satisfied by one way: when all modules at higher level are available, assemble them, otherwise use a new product to satisfy the demand and incur a penalty cost s_2, which can be equally viewed as a penalty cost due to shortage. Demand of a lower quality level ($l = 1$) has a more complicated reassembly decision making as follows: (1) when inventory levels of both modules at the lower level are positive, assemble them immediately. If not all modules at level $l = 1$ are available in stock, a decision needs to be made that whether to use the missing module from higher quality level $l = 2$ for assembly, or to use a new product to fulfil demand which incurs a cost s_1, but to save module of higher quality for future demand. Each assembly generates costs by using each module k at quality l at c_k^l. In addition, we assume $s_1 \geq s_2$ indicating that penalty cost of using a new product varies with the demand class. The objective is to determine an optimal policy that can tell how to fulfil stochastic demand for any given both intial system state (module inventories) and current stochastic returns information so as to minimize the time average cost.

4. Formulation

We formulate the problem as a Markov decision process (MDP) that works as follows: there is a state space, denoted S; and the state of the system $\mathbf{x} \in S$ is described by the on-hand inventory position of all types of modules at all different quality grades. The state of the system at time stage t can be described by the vector:

$$\mathbf{x}(t) = (x_1^1(t), \ldots, x_1^n(t), x_2^1(t), \ldots, x_2^n(t), \ldots, x_m^1(t), \ldots, x_m^n(t)),$$

where $x_k^l(t)$ is a non-negative integer denoting the on-hand inventory for module k with quality l at time t. The decision epochs correspond to the set of all demand arrival epochs. At each decision epoch, a policy specifies whether the reassembly station should assemble used modules from on-hand inventories or use a new product to satisfy the demand immediately. A Markovian policy, which is said to be stationary as it is independent of the history, specifies the action to be taken at all periods. In other words, action it takes at any decision epoch depends only on the system state but not the time. We let $V(\mathbf{x})$ denotes the relative cost of system in state $\mathbf{x}$, and g denotes the average cost per transition. Then using the average cost criterion and uniformization, we can write the following optimal equation

$$V_t(\mathbf{x}) + g = \frac{1}{\beta}\left[\begin{array}{l} h(x) + \sum_{k=1}^{m}\sum_{l=1}^{n}\mu_k p_l V_t(\mathbf{x} + \mathbf{e}_k^l) + \sum_{l=1}^{n-1}\lambda_l \min[V_t(\mathbf{x}^l - \mathbf{e}_{-k}^l, \mathbf{x}^{l+1} - \mathbf{e}_k^{l+1}) + c_l + c_{l+1}, \\ V_t(\mathbf{x}) + s_l] \cdot 1\{x_k^l = 0, x_{-k}^l \neq 0, x_k^{l+1} \neq 0\} + \sum_{l=1}^{n}\lambda_l[V_t(\mathbf{x}) + s_l] \cdot 1\{x_k^l = 0, x_k^{l+1} = 0\} \\ + \sum_{l=1}^{n}\lambda_l \min\{V_t(\mathbf{x}^l - \mathbf{e}) + 2c_l, V_t(x) + s_l\} \cdot 1\{\prod x_k^l \neq 0\} \end{array}\right] \quad (1)$$

where $\beta = \sum_{l=1}^{n} \lambda_l + \sum_{k=1}^{m} \mu_k$, $h(\mathbf{x}) = \sum_{k=1}^{m} \sum_{l=1}^{n} h_k^l x_k^l$, and $1\{\cdot\}$ denotes an indicator function. For all $k \in \{1,\ldots,m\}$ we define $\mathbf{e}_{-k}^l \triangleq [b_{k-1}\ 0\ b_{n-k}] \in \mathbb{R}^{1\times n}$, where $b_k \triangleq [1 \cdots 1] \in \mathbb{R}^{1\times k}$.

In Eq. (1), the first minimization operation corresponds to the decision of assembly by using a higher quality module to substitute the missing module. The second minimization operation corresponds to the rationing decision of using on-hand inventory or a new product to satisfy the current demand.

5. Structural optimal policy

The optimal reassembly policy determines whether it is cost-effective to fulfil a demand by assembling on-hand inventories or use new products to save inventory for future demand. The policy would affect the long run cost of the remanufacturer.

Based on the numerical experiments using the relative value iteration algorithm [15], we conjecture that there exists a *stationary optimal policy* that is characterized as a state-dependent threshold policy. We describe the optimal policy for lower quality and higher quality demand respectively as below.

(1) For a lower quality demand ($l = 1$), if all modules at $l = 1$ are available, i.e., $\Pi_{k=1}^{m} x_k^l \neq 0$, it is always optimal to assemble them to fulfil the current demand. If module k with lower quality is not available ($x_k^l = 0$), then it is optimal to assemble all on-hand modules (except module k) with quality $l = 1$ with a module k with higher quality $l = 2$ if all the following conditions are satisfied:
   a) Its substituted module inventory level exceed a rationing level $\gamma_{k,l}^*(\tilde{\mathbf{x}})$, i.e., $x_{l+1}^k > \gamma_{k,l+1}^*(\tilde{\mathbf{x}})$
   b) Each on-hand module inventory level except module k of lower quality falls below $\rho_{-k,l}^*(\tilde{\mathbf{x}})$, i.e., $x_{-k}^l < \rho_{k,l}^*(\tilde{\mathbf{x}})$
   c) Each on-hand module inventory level except module k of lower quality falls below $\rho_{-k,l+1}^*(\tilde{\mathbf{x}})$, i.e., $x_{-k}^{l+1} < \rho_{k,l+1}^*(\tilde{\mathbf{x}})$

   Otherwise, refuse to reassemble and use a new product to fulfil the demand.
(2) For a higher quality demand ($l = 2$), it is optimal to assemble the modules from the on-hand inventory if the following two conditions hold:
   d) Inventory level for each lower quality modules ($l = 1$) falls below $\rho_{k,l-1}^*(\tilde{\mathbf{x}})$, i.e., $x_k^{l-1} < \rho_{k,l-1}^*(\tilde{\mathbf{x}})$, for all k. $k = 1, 2$.
   e) Inventory level for each higher quality modules exceeds a rationing level $\gamma_{k,l}^*(\tilde{\mathbf{x}})$, i.e., $x_k^l > \gamma_{k,l}^*(\tilde{\mathbf{x}})$, for all $k = 1, 2$.

Otherwise, use a new product to fulfil demand.

The optimal policy first states that demand of a lower quality grade should always be satisfied by reassembling on-hand lower quality modules if there is no missing lower quality module in stock. Otherwise, decisions on whether to use substituted higher quality modules should be made depending on inventory level of all other modules (conditions (a)–(c)). The latter part of the optimal policy states that a demand of a higher quality grade will be fulfilled by reassembly only if the inventory level of each higher

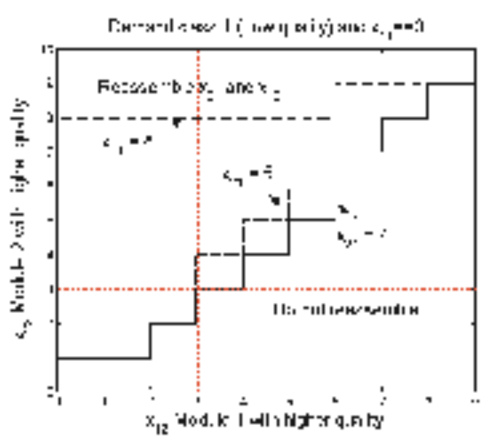

Fig. 2. The structure of reassemble-to-order policy (demand class 1).

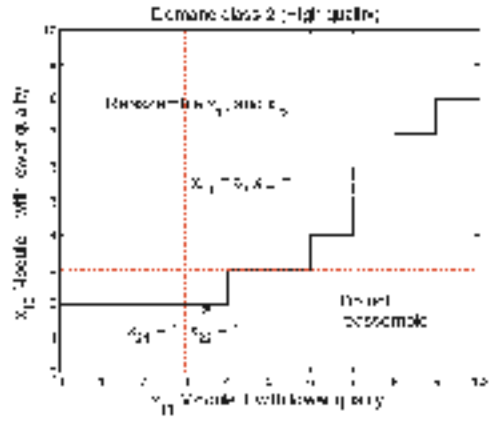

Fig. 3. The structure of reassemble-to-order policy (demand class 2).

quality module exceeds a critical value, and each the lower quality module inventory is below a critical amount.

6. Numerical studies

We illustrate the structure of the optimal policy using an example problem with two modules and two quality grades (two demand classes) with the following setting of parameters: $\lambda_1 = \lambda_2 = 0.6$, $\mu_1 = \mu_2 = 2$, $p_1 = p_2 = 0.5$, $c_1 = 0.3$, $c_2 = 0.5$, $s_1 = 10$, $s_2 = 8$, $h_1^1 = h_2^1 = 4$, $h_1^2 = h_2^2 = 6$. We let x_{11} and x_{21} denote module 1 and 2 with lower quality and x_{12} and x_{22} denote module 1 and 2 with lower quality. The optimal policy is shown in Figs. 2 and 3 for higher and lower quality demand class, respectively. In Fig. 2, the black solid, dashed and dotted curves are results of two-dimensional cuts on the region-switching hypersurfaces at three different x_{21} values. Each switching curve divides the state space into two regions, i.e., reassemble region (above) and do not reassemble region (below).

Fig. 3 shows the optimal policy associated with a higher quality demand ($l = 2$), where the black solid and dashed curves represent the switching curves at two different values of the lower/higher quality module 2 inventory (x_{21}, x_{22}). Furthermore, it is observed that all the switching curves are monontically increasing curves. As expected, higher quality modules would be saved for later assembly for a higher quality demand prior to the choice that it is immediately used to substitute a missing lower module.

6.1. Heuristics for reassembly control

The state-dependent optimal policy structure determined in the previous section is fairly complex. The reassembly and inventory control decisions need to be made across the inventory level of each module with various quality grades. As the number of modules gets larger or the number of quality grades gets larger, the size of state space will grow exponentially. Hence, the computation

Table 1
Comparison of optimal policy and heuristic policy.

	Arrival rates				Cost parameters		Average costs		
	μ_1	μ_2	λ_1	λ_2	s_1	s_2	Optimal	Heuristic	% Gap
1	3.0	3.0	0.5	0.5	10	8	221.79	227.94	2.77
2	2.0	4.0	0.5	0.5	10	8	129.79	134.99	4.01
3	4.0	2.0	0.5	0.5	10	8	186.79	208.65	11.70
4	4.0	4.0	0.5	0.5	10	8	164.01	169.29	3.22
5	3.0	3.0	0.7	0.7	10	8	234.26	241.52	3.10
6	2.0	4.0	0.7	0.7	10	8	136.6	144.03	5.44
7	4.0	2.0	0.7	0.7	10	8	206.9	236.59	14.35
8	4.0	4.0	0.7	0.7	10	8	173.01	179.44	3.72
9	3.0	3.0	0.5	0.5	9	5	215.32	220.01	2.18
10	2.0	4.0	0.5	0.5	9	5	123.67	129.16	4.44
11	4.0	2.0	0.5	0.5	9	5	182.89	200.95	9.87
12	4.0	4.0	0.5	0.5	9	5	159.86	163.16	2.06
13	3.0	3.0	0.7	0.7	9	5	223.79	230.16	2.85
14	2.0	4.0	0.7	0.7	9	5	127.8	135.77	6.24
15	4.0	2.0	0.7	0.7	9	5	200.79	225.05	12.08
16	4.0	4.0	0.7	0.7	9	5	166.41	170.77	2.62

of the optimal policy and makes their actual implementation rather problematic because of the "curses of dimensionality". In this section, we propose a heuristic approach to provide practical implementable policies and compare its performance to that of the optimal policy in the previous sections. The heuristics consists of controlling reassembling used modules of each quality level independently of each other with a set of fixed rationing levels, denoted by a matrix $\boldsymbol{\gamma}_l = (\gamma_{1,l}, \gamma_{2,l}, \ldots, \gamma_{m,l})$ and a set of fixed acceptance thresholds $\boldsymbol{\rho}_l = (\rho_{1,l}, \rho_{2,l}, \ldots, \rho_{m,l})$ for each grade l, $l = 1, \ldots, n$. The average cost of the heuristic can be obtained for a reassemble-to-order system with multiple returned module types with various quality grades via the following dynamic programming iterations:

$$V_{t+1}(\mathbf{x}) = [h(\mathbf{x}) + \sum_{l=1}^{n-1} \lambda_l T V_t(\mathbf{x}) + R V_t(\mathbf{x})]/\beta,$$

where T and R are operators associated with re-assembly decisions upon an incoming demand arrival and the system state changes due to an incoming module returns, respectively.

$$TV(\mathbf{x}) = \begin{cases} V(\mathbf{x}^l - \mathbf{e}^l_{-k}, \mathbf{x}^{l-1} - \mathbf{e}^{l-1}_k) + c_l + c_{l+1}, & \text{if } x^l_k = 0, \mathbf{x}^{l+1}_k \geq \boldsymbol{\gamma}_{l+1}, \mathbf{x}^{l-1}_{-k} < \boldsymbol{\rho}_{l+1} \\ V(\mathbf{x}^l - \mathbf{e}) + 2c_l, & \text{if } \prod_{k=1}^{m} x^l_k \neq 0, \mathbf{x}^l \geq \boldsymbol{\gamma}_l, \mathbf{x}^{l-1} < \boldsymbol{\rho}_l \\ V(\mathbf{x}) + s_l, & \text{otherwise} \end{cases} \quad (2)$$

$RV(\mathbf{x}) = \sum_{k=1}^{m} \sum_{l=1}^{n} \mu_k p_l V_t(\mathbf{x} + \mathbf{e}^l_k)$ with the inequality of $\boldsymbol{\gamma}_l$ and $\boldsymbol{\rho}_l$ taken module-wise.

We perform a 10 by 4 dimensional search to obtain the best sets of $\boldsymbol{\gamma}_l$ and $\boldsymbol{\rho}_l$ values and the average cost associated with these values. The optimal thresholds are selected when they result in the lowest cost. We show the results of the heuristic policy in the same figures in Section 4. The red dotted lines in both Figs. 2 and 3 are the *state-independent* switching curves that separate the optimal decision regions. In both figures, the upper-left regions of both figures are the "reassembly" region; all other three quarters correspond to "do not reassembly" region.

To compare the performance of the heuristic policy versus the optimal assembly policy, we numerically tested 16 examples where different arrival and demand scenarios are systematically set (Table 1). The average profit per unit time for the optimal and the heuristic policy are obtained together with the percentage gap of the heuristics from the optimal.

7. Conclusions and future research

In this paper, we considered optimal decisions on reassembly of modular product in a remanufacturing system with both supply and demand uncertainties in terms of timing and quality. The structures of optimal policy were analyzed. We have also proposed a state-independent heuristic approach and compare its performance with the optimal policy. It is shown that the heuristic policy performs effectively in most of the cases.

Future extension to this work is to test different quality variance of the returned products to further investigate its impact to the optimal decision and the effectiveness of the optimal policy.

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