Feature Presentation

RESOLVER IN DIGITAL CONTROL LOOP

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ABSTRACT

A digital control loop based on a resolver as feedback device is considered. This technique is popular in NC systems of machine tools. The control circuit transforms a train of pulses generated by a digital interpolator into a phase-modulated command signal and compares it with a phase-modulated feedback signal. The resulting phase-difference is the drive signal to the motor. The principle of the loop is explained, and a mathematical analysis is presented.

INTRODUCTION

One of the most popular techniques in numerical control (NC) of machine tools is based on a resolver as feedback device. A typical NC unit comprises several controlled axes, each of them equipped with a resolver. All control circuits of these axes are identical, hence an analysis involving a single control loop should suffice.

In NC literature it is hard to find a complete analysis of a control loop which contains a resolver. A control circuit is explained in [1], but we were unable to find a source presenting a mathematical analysis of the loop.

In NC systems, a resolver is coupled to each leadscrew of the machine tool and provides a signal indicating the angular position of the latter. Usually the resolver is geared for one revolution per 0.1 inch of linear motion. The practical accuracy of a resolver is one-thousandth, which means a precision level of 0.0001 inch. This unit is also the system resolution, and will be referred to henceforth as the basic length unit, i.e., 1 BLU = 0.0001 inch.

The resolver-based control loop may be applied to other digitally controlled systems. As many engineers are apparently unfamiliar with this technique, we shall try to remedy this situation by providing the general background in the first section, and explain the control loop in the second. The loop analysis is presented in the last two sections.

THE RESOLVER

The resolver is a rotary device consisting of two rotor and two stator windings, each pair in a mutually perpendicular arrangement as shown in Figure 1. When the resolver serves as a feedback measuring device, one only of the rotor windings is used. The stator windings are fed by sine-wave signals in quadrature, v_1 and v_2, equal in amplitude, namely:

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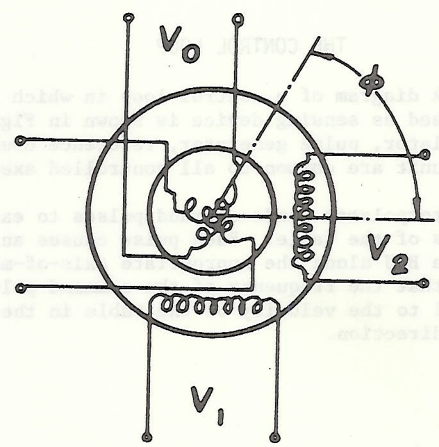

Figure 1 : Resolver.

$$v_1(t) = V_a \sin\omega_o t \tag{1a}$$

$$v_2(t) = V_a \cos\omega_o t \tag{1b}$$

It should be noted that in order to maintain the required accuracy of one-thousandth of a revolution, the quadrature shift of the reference signal must be within a tolerance of 0.36 degrees.

The output of the resolver is the rotor signal, which is a function of the rotating angle and is obtained by inductive coupling between stator and rotor. The rotor output voltage, v_o, consists of two components:

$$v_o(t) = n[v_1(t)\cos\phi + v_2(t)\sin\phi] \tag{2}$$

where n is a constant dependent on the rotor/stator turns ratio. Substituting v_1 and v_2 as above, we have:

$$v_o(t) = nV_a(\sin\omega_o t\cos\phi + \cos\omega_o t\sin\phi) \tag{3}$$

or, denoting $nV_a = V$

$$v_o(t) = V\sin(\omega_o t+\phi) \tag{4}$$

The phase angle ϕ depends on the angular position of the rotor axis. Note that if the rotor is rotated through ϕ mechanical degrees, its output voltage is shifted by ϕ electrical degrees.

If the rotor rotates continuously with angular velocity $\omega(t)$, we may substitute in Eq. (4)

$$\phi(t) = \int_0^t \omega(t)dt \tag{5}$$

For a constant velocity ω, the steady-state feedback signal is:

$$v_o(t) = V\sin[(\omega+\omega_o)t + \phi_o] \tag{6}$$

where ϕ_o is the cumulative angle from t=0 to steady state. From the last equation we see that the feedback has the form of a phase-modulated wave, which is the basis of the control loop.

THE CONTROL LOOP

A block diagram of a control loop in which the resolver is used as sensing device is shown in Figure 2. The interpolator, pulse generator, reference counter, and filter unit are common to all controlled axes.

The interpolator sends command pulses to each controlled axis of the table. Each pulse causes an advance of one BLU along the appropriate axis-of-motion. This means that the frequency of the command pulses is proportional to the velocity of the table in the corresponding direction.

Positional control is effected by position counters. Each axis-of-motion is provided with a counter to which the required incremental distance is fed from the perforated tape. Each time a command pulse is sent by the interpolator, the contents of the appropriate counter are reduced by one unit. On reaching the zero position, the position counter blocks the command pulses.

The reference counter divides the clock frequency, emitted by the pulse generator, by a factor of 1000 and provides (by digital techniques) two square-wave signals in exact quadrature, characterized as the reference signals. The latter are converted by low-pass filters into sine-wave signals, which are used as excitation voltages for the stator windings.

The rotor output voltage is fed through a wave shaping circuit (Schmitt trigger) to a phase comparator, or discriminator, where it is compared with a command signal. The phase difference between the command and feedback signals is converted into a DC voltage, passed through a single-pole low-pass filter, amplified and used to drive the motor. The phase-comparator operation may be based on the principle of an up-down counter, which counts the falling edges of the command (up) and feedback (down) signals. A counter with i binary stages can hold an error of $\pm(2^{i-1} - 1)$ cycles. In many commercial systems the maximum error is ±1 cycle, i.e., 360 degrees, which is equivalent to ±0.1 inch.

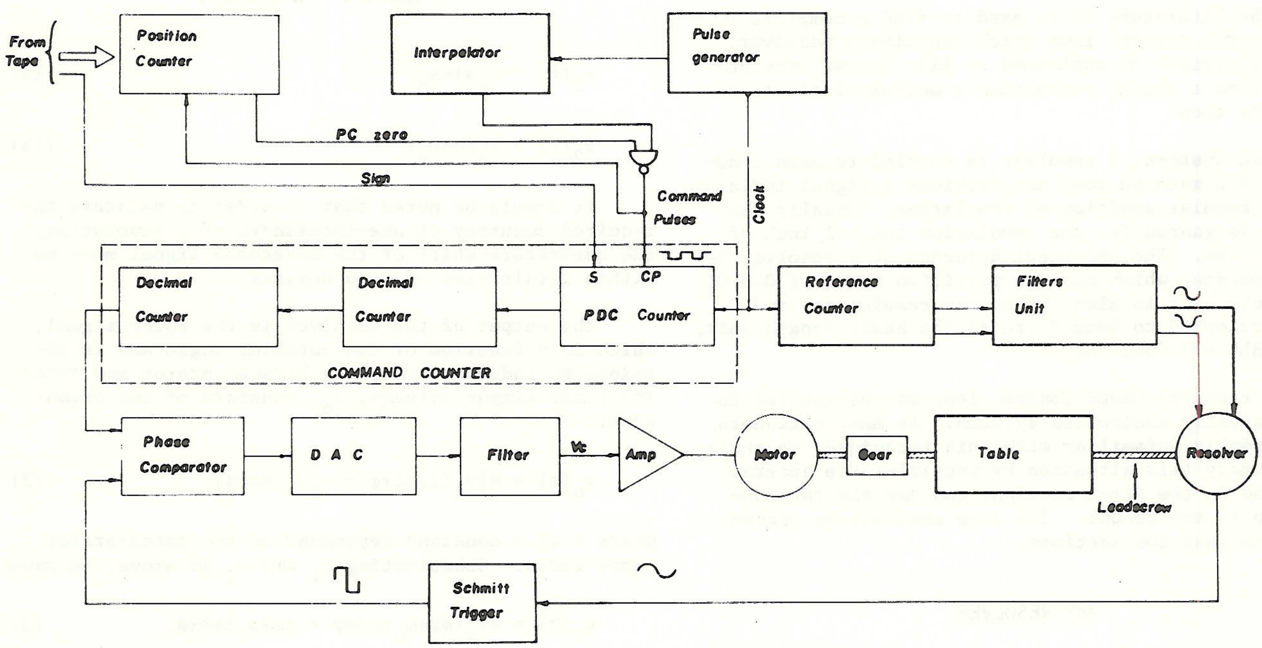

Figure 2 : Block diagram of the control system.

The command signal is produced by a command counter which consists of two fixed decimal counters and a programmable-division-factor counter (PDC). the PDC is fed by the command pulses (the CP input), a clock (typical frequency 2.5MHz), and a logic signal called the "sign" which indicates the required direction of motion. The PDC converts the command pulses into a phase-modulated signal. Each command pulse causes a phase shift of 0.001 cycle (0.36 degree) with respect to the reference signal. The phase shift is forward or backward (i.e., leading or lagging) depending on the sign logic level. The division factor, N, of the PDC also varies in accordance with the logic level at its CP and S (sign) inputs, as shown in Table 1.

Table 1: PDC Division Factor

CP	S	N
0	0	5
0	1	∞*
1	0	10
1	1	10

* ∞ denotes a no-count condition.

So long as the CP input is at the "1" level, the PDC acts as a decimal counter and its input clock frequency (2.5MHz) is divided by 10. Therefore, so long as the resolver is at rest, the feedback and command signals have the same frequency (2.5KHz) and are exactly in phase; as a result, the velocity-command signal (VC) is zero and the motor also remains at stand still.

As was explained, in the case CP=1, a single cycle of the command signal is emitted by the command counter for every 1000 clock pulses. Assume now that S=1 and CP=0 for an interval covering a single clock pulse (400nsec when a 2.5MHz clock is used). According to Table 1 for S=1, the clock pulse sent in this interval is not counted (N=∞), so that 1001 clock pulses are required to effect one cycle of the command signal. This means that the falling edge of the command signal lags by 1/1000 of a cycle compared with its previous state.

To illustrate a lead case, assume that CP=0 for an interval of 10 clock pulses (i.e., 4000nsec), and that S=0. In this interval the PDC sends only 2 pulses (since N=5); for the following 980 pulses the PDC emits 98 pulses (since CP is reset to 1). Altogether the PDC sends 100 pulses for 990 clock pulses. In other words, 990 clock pulses are required to effect one cycle of the command signal, which means that the command signal leads by 10/1000 of a cycle. The CP input is fed by negative pulses of 400 nanoseconds width, each of which caused advance along the appropriate axis-of-motion by one BLU, i.e., 0.0001 inch. At S=0 these pulses make the command signal lead the reference signal in phase and the motor rotates in a certain direction, similarly, at S=1 the motor rotates in the opposite direction. When standstill is required, the CP input is at the "1" level and the PDC divides by 10.

FREQUENCY RANGE

When command pulses are sent through the CP input, the average division factor of the PDC varies according to their frequency, as explained below.

The duration of each negative pulse is T=1/f, f being the clock frequency (i.e., 2.5MHz). The highest possible frequency of the command pulses is f/2. If n pulses are sent in the interval to the CP input, the average frequency of the command pulses is n/t pps. We denote the ratio of the two frequencies by p:

$$p = \frac{n/t}{f} = n\frac{T}{t} \tag{7}$$

Since the n pulses are negative, the CP input is at the "0" level in the interval:

$$nT = pt \tag{8}$$

during which the PDC count depends on the S input level. The interval in which the CP input is the "1" level is:

$$t - nT = t(1-p) \tag{9}$$

during which the PDC operates as a decimal counter.

The average output frequency from the PDC (while the motor rotates) is obtainable from Table 1 and Eqs. (8) and (9). For S=1 the frequency is:

$$\frac{1}{t}\left[\frac{f}{10} \times (1-p)t + 0 \times pt\right] = \frac{f}{10}(1-p) \tag{10}$$

For S=0 the frequency is:

$$\frac{1}{t}\left[\frac{f}{10} \times (1-p)t + \frac{f}{5} \times pt\right] = \frac{f}{10}(1+p) \tag{11}$$

The command counter comprises two additional decimal counters which divide the PDC output by a factor of 100. Therefore, the range of the command signal frequency f_c, which is the output of the command counter, may be:

$$\frac{f}{1000}(1-p) \leqslant f_c \leqslant \frac{f}{1000}(1+p) \tag{12}$$

Introducing the definitions:

$$f_r = \frac{f}{1000} \quad ; \quad f_o = pf_r$$

f_r being the reference frequency and f_o 1/1000 of the frequency of the command pulses entering the PDC, Eq. (12) becomes:

$$f_r - f_o \leqslant f_c \leqslant f_r + f_o \tag{13}$$

The range of the factor p is $0<p<\frac{1}{2}$, and therefore, theoretically, the maximum f_o is $f_r/2$, but in practical NC systems this maximum does not exceed $f_r/100$. For example, a requirement of a maximal feed-rate of 150ipm in a system with a BLU of 0.0001 inch means a maximal feed-rate of 25000 BLU/sec, which in turn dictates f_o=25cps. A reference frequency of 2500cps in such a system yields p=1/100, i.e., the frequency of the command signal may vary from 2475 through 2525cps, as can be seen from Eq. (13).

MATHEMATICAL ANALYSIS

Although the control loop is of the digital type, the simplest way to analyze it is to assume linearity and apply the Laplace transform.

Figure 3 shows a simplified block diagram of the loop. The frequency of the excitation voltages to the resolver stators is f_r. The command signal for a right rotation is a square wave with frequency (f_r+f_o). When

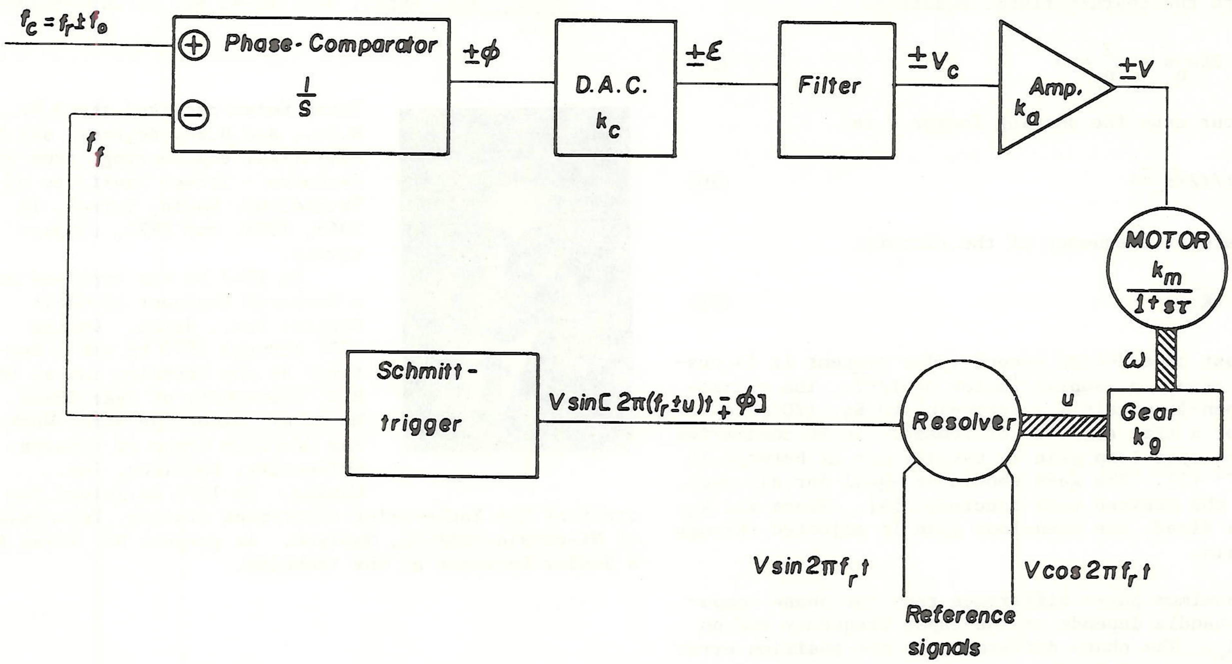

Figure 3 : Simplified block diagram of the control loop.

a constant speed is required, this frequency is constant as well. At steady state both the command and feedback signals have the same frequency, but the latter lags the former in phase by ϕ degrees. This phase difference is converted into a voltage, which in turn drives the motor so that the resolver rotor runs at f_o revolutions per second. The speed of the motor can thus be controlled by varying the frequency f_o. Under a command signal with frequency $(f_r - f_o)$, the feedback signal leads the command signal and the motor rotates in the opposite direction.

To make the loop analysis possible the transfer function of the phase comparator must be defined. This circuit receives the command and feedback frequencies and yields a phase difference ϕ, thus acting as an integrator; its transfer function is:

$$\frac{\phi}{f_c - f_f} = \frac{1}{s} \tag{14}$$

Note that frequencies are measured in cps and ϕ in cycles. The open-loop gain is:

$$K_v = \frac{f_o}{\phi}\left[\frac{1}{\text{sec}}\right] \tag{15}$$

and is the product of the following gains:

(1) The DAC gain k_c (in volt/cycle).

(2) The amplifier gain k_a.

(3) The gear ratio k_g.

(4) The motor gain k_m, in turn defined by its transfer function:

$$\frac{\omega}{v} = \frac{k_m}{1+s\tau}\left[\frac{\text{rev/sec}}{\text{volt}}\right] \tag{16}$$

where τ is the mechanical time constant of the motor coupled to the machine slides. A typical time constant of an NC drive is around 20 msec.

In many NC systems a typical drive includes an internal loop with a tachogenerator as an additional feedback element, but Eq. (16) is (albest approximately) valid for the transfer function of this loop, as was shown in [2].

The filter is a single-pole low-pass filter with unity gain and a time-constant which is much smaller than that of the motor (for stability reasons), and need not be considered in this analysis. Hence, the transfer function of the open loop is:

$$G(s) = \frac{K_v}{s(1+s\tau)} \tag{17}$$

and for the closed loop:

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K_v/\tau}{s^2 + s/\tau + K_v/\tau} \qquad (18)$$

Accordingly, the control loop is a second-order servo system with the characteristic equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \qquad (19)$$

where in our case the damping factor ξ is:

$$\xi = 1/(2\sqrt{K_v \tau}) \qquad (20)$$

and the natural frequency of the circuit:

$$\omega_n = \sqrt{K_v/\tau} \qquad (21)$$

In most closed-loop second-order systems it is customary to choose a damping factor of $1/\sqrt{2}$. The corresponding open-loop gain K_v, according to Eq. (20), is $25sec^{-1}$ for a time constant of 20msec. In NC contouring systems the open-loop gain is usually set at between 15 and $35sec^{-1}$ [3]. The gain should be equal for all axes, to secure the desired path accuracy, [4]. Since k_a, k_g, and k_m are fixed, the open-loop gain is adjusted through the DAC gain.

The maximum phase difference that the phase comparator can handle depends on the input frequency and on the gain K_v. The phase difference is the position error between command and feedback and is given by the equation:

$$\phi(s) = \frac{f_o(s)}{\tau s^2 + s + K_v} \qquad (22)$$

For a constant input frequency at steady state the phase-difference is:

$$\phi = f_o/K_v \qquad (23)$$

In a system with maximum f_o 20cps and gain $25sec^{-1}$, Eq. (23) results in ϕ=0.8 cycle. The capacity of the phase comparator in this case is ±1 cycle, which is usually equivalent to 1000 pulses, or ±0.1 inch of linear motion.

CONCLUSION

The paper describes the operation of a digital control loop based on a resolver as feedback device. The accompanying mathematical analysis, which involves stability considerations as well as frequency calculations, can be of substantial use in design problems.

We hope that this presentation will induce designers to apply this technique wherever the main requirement is identical to that of NC systems: simultaneous operation of several axes-of-motion.

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