MATHEMATICAL MODEL FOR THE FLANK WEAR WHILE TURNING STEEL WITH CARBIDE TOOLS

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Y. Koren and E. Lenz

reprint from

Proceedings of the CIRP Seminars on

MANUFACTURING SYSTEMS

Vol. 1 No. 2 1972

The paper was presented at the CIRP Second International Seminar on Manufacturing Systems, Trondheim, June 1970.

MATHEMATICAL MODEL FOR THE FLANK WEAR WHILE TURNING STEEL WITH CARBIDE TOOLS*

Y. KOREN AND E. LENZ**

ABSTRACT

Design of optimal control for a machine tool necessitates a mathematical model of the machining process, in view of the unfeasibility of carrying out all tests on the machine. A model for steel turning with a carbide tool was developed accordingly, yielding the relation between the process parameters (cutting speed, feed depth of cut and rake angle) on the one hand, and tool wear (the unknown function in the performance index of the optimization) on the other. The internal variables of the model are tool temperature and the vertical cutting force. Results are analysed and compared with those known from literature.

NOTATION

F	— vertical cutting force
C_z, C_y, C_v, c	— coefficients in force formula
v	— cutting speed
θ	— temperature
θ_e	— final temperature
T	— tool life
B	— wear land
a	— depth of cut
s	— feed
t	— time
γ	— rake angle
τ_0	— force time constant
τ_e	— temperature time constant
p	— Laplace transformation complex-variable

* This paper is part of Dr. Y. Koren's D. Sc. Thesis; Professors J. Ben-Uri, E. Lenz, and D. Graupe were his supervisors.

** The authors are members of the Faculty of Mechanical Engineering, Technion — Israel Institute of Technology, Haifa, Israel.

INTRODUCTION

The study of machine tool optimization comprises two stages:

1. identification of the system, including the machine behavior (tool wear, forces, temperatures, vibration, etc.);
2. the optimization proper, the problem consisting in solution of controllable process conditions (cutting speed, feed, depth of cut, tool angles) with a view to the required product quality and to minimum cost.

The identification stage may consist in developing either the deterministic or a probabilistic model. The present study yielded a model of the former type, capable of transformation into the latter type by incorporating a noise source. The model is partial, and interrelates the cutting force, tool temperature and tool wear.

The wear land, rather than the crater, was taken as wear criterion. A comprehensive study of the wear land by Müller[11], yielded four expressions for its time behavior:

$$B = \alpha_1 \sqrt{t} \tag{1}$$

$$B = \alpha_2 t^x + \beta_2 \qquad 0.5 < x < 1 \tag{2}$$

$$B = \alpha_3 t + \beta_3[1 - \exp(-\gamma_3 t)] \tag{3}$$

$$B = \alpha_4 ln(1 + \beta_4 t) + \gamma_4 t^n \tag{4}$$

Where α_1, α_2, α_3, α_4, β_2, β_3, β_4, γ_3, γ_4, are constants.

Although the last two expressions proved to be good approximations to experimental curves, Müller's work offers no physical interpretation. The present paper proposes a formula which, simplified, leads to eq. (3).

CUTTING FORCE

The proposed model considers only the main (vertical) cutting force. Its dependence on the feed rate and the chip depth is usually given by:

$$F = C_z s^u a \tag{5}$$

where C_z depends on the steel type. The exponent u, according to various sources[1, 2, 12], ranges from 0.74 to 0.82, with an average of 0.76.

The author found that a more accurate expression is:

$$F = C_z s^u a - ca \tag{6}$$

where c is constant equal about 5% of C_z. In the example shown in Fig. 1, the force is plotted against the feed during longitudinal turning of 2 Kh 13 steel, tool 20 A-0, $a = 4$ mm, $v = 110$ m/min (data reproduced from Ref. [2], Fig. 125). It is seen that the relationship $F = 700 s^{0.76} - 35$ is in better agreement with measurement results than $F = 665 s^{0.82}$.

In addition to the factors appearing in eq. (5), the cutting force depends also on the back rake angle γ and the cutting speed v. It is known that the force decreases with increasing γ. According to charts 1 to 5 in Ref. [2], the decrease is given by:

$$F = F_{o\gamma}(1 - C_\gamma \gamma) \tag{7}$$

where $F_{o\gamma}$ is the main force for $\gamma = 0$.

If γ is in degrees, C_γ ranges from 0.009 to 0.011 according to the steel type, the average being $C_\gamma = 0.01$.

The relation between the force and the cutting speed is more complicated. Initially, the force increases with the speed up to the level of 20 m/min and subsequently decreases; above 60 m/min (carbide tools) this decrease may be regarded as linear, with a slope which is linear with cutting depth:

$$F = F_{ov} - C_v va \tag{8}$$

where F_{ov} and C_v are constants.

If F is in kp and v in m/min, C_v is about 0.01.

Eqs. (6), (7) and (8) may be combined into a general expression for the vertical cutting force:

$$F = [C_z s^n(1 - C_\gamma \gamma) - c - C_v v]a \tag{9}$$

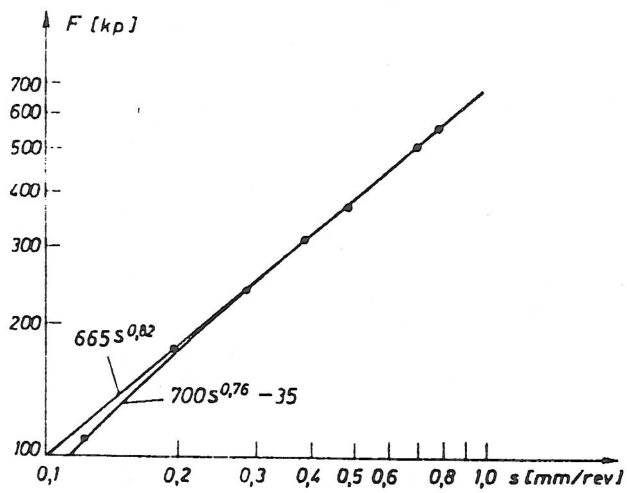

Figure 1. Force vs. feed rate (conventional and new formuloe)

TEMPERATURE

It is assumed that the tool temperature obeys the following differential equation:

$$\tau_e \dot{\theta} + \theta = \theta_e \tag{10}$$

where θ_e is the final temperature and τ_e the time constant of the process. According to Refs. (1) and (3), the time constant ranges from 10^{-2} to 10^{-4} sec., while θ_e depends on the cutting speed, force, feed, depth of cut and workpiece material, or in the most general form:

$$\theta_e = K_T F^r v^x s^y a^z \tag{11}$$

According to Refs. [1], [2], [5], [6] and [12], $r = 1$ and x ranges from 0.4 and 0.5, i. e. an average of $x = 0.45$. The power y is variously given as -0.78[1], -0.5[6],

-0.55[12], and as dependent on v[2]; its average was calculated as -0.55. The value of z also varies between the sources, and its average was calculated as -0.95. The formula thus becomes:

$$\theta_e = K_T v^{0.45} s^{-0.55} a^{-0.95} F \tag{12}$$

A typical value of K_T (with F given in kp, v in m/min, s in mm per revolution, a in mm and θ_e in degrees Centigrade) is 0.5.

WEAR LAND

The wear characteristic comprises three stages:

1. An initial stage with high wear rate, relatively short compared with tool life.
2. A near-linear stage.
3. A final high-rate stage, representing tool failure.

Tool wear was affected by three principal mechanisms: adhesion, abrasion and diffusion.

Adhesion is the wear due to formation of pressure welding points and their subsequent destruction. When destruction is by shear below the interface, a wear particle is shifted. This mechanism is confined to low cutting temperatures[10], and is thus without interest in the present discussion.

Abrasion, or removal of asperities of one surface by the material of the other, is proportional to the stress on the tool. At the beginning, the contact area between workpiece and tool is small, so that the stress is high and the wear is intensified accordingly, until after a certain time both the contact area and stress become constant. This interval is inversely proportional to the cutting speed. (The abrasion wear (B_F) continues to increase slowly throughout the cutting process due to the increase in the force, as explained later.)

By the above considerations, the direct effect of the cutting force on the wear is confined to the initial stage, and the following differential equation may be assumed as valid:

$$B_F + \tau_o \dot{B}_F = K_F \frac{F_n}{s \cdot a} \tag{13}$$

where F_n is the normal force and $\tau_o = l_o/v$.

From Merchant's model it is known that:

$$F_n = F\cos\gamma - F\sin\gamma_n \cong F\cos\gamma \tag{14}$$

whence

$$B_F + \frac{l_o}{v}\dot{B}_F = K_F \cos\gamma \frac{F}{s \cdot a} \tag{15}$$

where the length constant l_o is about 300 to 700 meters and the order of magnitude of K_F is 5×10^{-4} mm^3/kp[7].

As for *diffusion*, its rate is an exponential function of temperature; at the same time, the corresponding rate of material transport varies as the square root of the cutting speed (see App. I), so that the wear rate due to diffusion is:

$$\dot{B}_D = K_D \sqrt{v} \exp(-A/\theta^\circ K) \tag{16}$$

where K_D and A are constants depending on the types of carbide and steel. In the example shown in Fig. 2 (depth of diffusion vs. temperature) the curve $l_D = 110 \exp(-10\,000/\theta^\circ K)$ corresponds to experimental results drawn from Ref. [10].

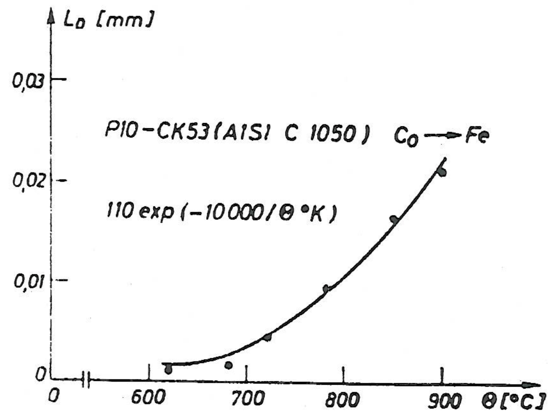

Figure 2 Depth of diffusion vs. temperature.
P-10 carbide: AISI-C 1050 stell

FORCE VS. WEAR

According to Ref. [8], we have for carbide tool machining of steel:

$$F = F_o + C_w aB \tag{17}$$

This formula was confirmed by linear regression, with correlation coefficients exceeding 0.9: C_w, as found from Fig. 3 in the above reference, is 50 kp/mm² for 0.45% carbon steel, a P-20 carbide tool and 0.2 mm/rev. feed.

According to Ref. [2], the cutting force comprises two components:

$$F = F_o + F' \tag{18}$$

with the force increment due to the wear given by:

$$F' = q'\mu(a/\sin\phi + r\tan\phi/2 + s)\cdot B \tag{19}$$

where r — radius of cutting tool corner; ϕ — main cutting adge angle; μ — coefficient of friction, q' — specific normal load on clearance face.

Figs. 322 and 329 in Ref. [2] (steel and carbide-tool data) show that μ increases and q' decreases (both slightly) with decreasing a, so that the product $q'\mu$ may be taken as constant.

Defining:

$$C_w = q'\mu/\sin\phi \tag{20}$$

and taking the average ϕ as 45°, we obtain:

$$F' = C_w(a + 0.3r + 0.7s)\cdot B \tag{21}$$

i. e. eq. (17) is theoretically justified for $a \geq 1$ mm, $r < 0.5$ mm, $a > 7s$. An average value for C_w according to Ref. [2] is 30 kp/mm².

Further confiirmation of eq. (17), from a different point of view, is given in Ref. [9].

MATHEMATICAL MODEL

Eqs. (9), (10), (12) and (15)—(17) yielded the block diagram of the mathematical model as shown in Fig. 3, with a noise generator incorporated to represent inhomogeneity of the workpiece.

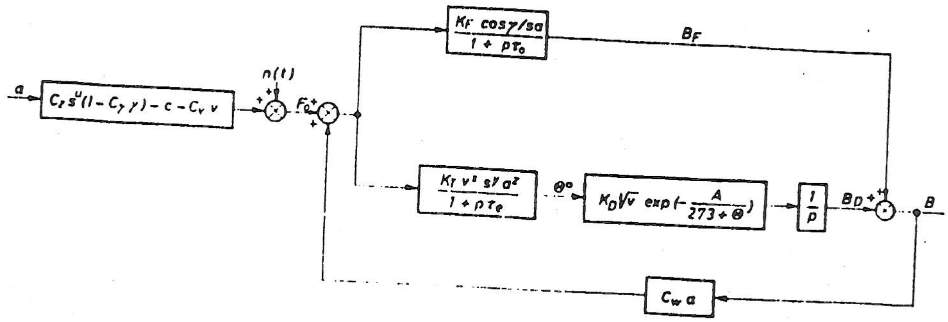

Figure 3. Block diagram of model

The model was simulated on a computer for the following data:

$C_z(1 - C_\gamma \gamma) = 200$	$C_w = 30$
$K_F \cos \gamma = 510^{-1}$	$u = 0.76$
$C_v = 0.01$	$x = 0.45$
$c = 6.5$	$y = -0.55$
$K_T = 0.55$	$z = -0.95$
$K_D = 20$	$\tau_e = 0$ (so $\theta = \theta_e$)
$A = 10.000$	$\tau_o = 500$

Fig. 4 gives the wear plotted against time, with the speed as parameter (range 70—180 m/min), $s = 0.25$ mm/rev., $a = 2.5$ mm. Simulation results are seen to agree

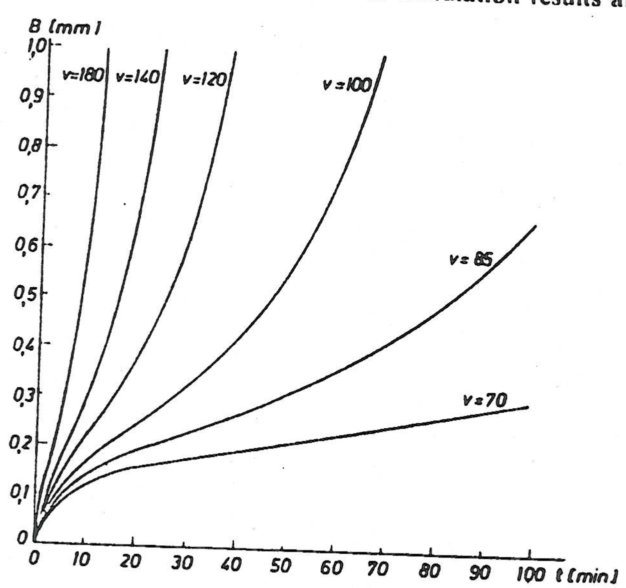

Figure 4. Wear curves — computer simulation
s 0.25 mm/rev; a 2.5 mm

with practical curves. If the limit of the tool life is set at 0.75 mm wear, its corresponding value is 60 min at a speed of 100 m/min.

Fig. 5 shows the time dependence of the abrasion and diffusion components, B_F and B_D. The former is seen to predominate in the early stages, and the latter in the final stages.

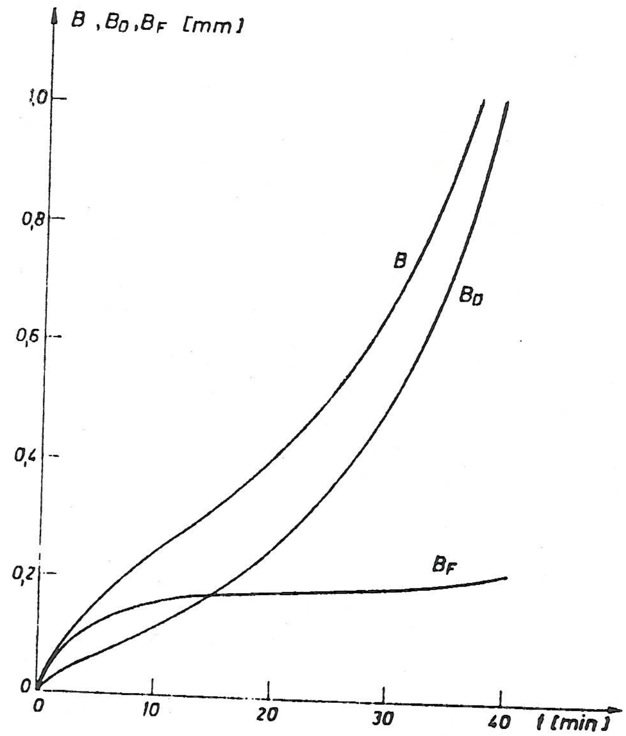

Figure 5. Diffusion and adrasion wear vs. cutting time

For a wear land of 0.75 mm, cutting speed (Fig. 6a), feed (Fig. 6b) and depth of cut (Fig. 6c) were plotted against tool life in a logarithmic scale, yielding straight lines, as is the case in practice. The lines in Fig. 6a have almost the same slope, corresponding to the formula:

$$v^{3.17}T = C_1 \tag{22}$$

known as Taylor's equation.

The lines in Fig. 6b are also almost parallel, and we obtain:

$$s^{1.14}T = C_2 \tag{23}$$

From Fig. 6c it is seen that:

$$a^{0.294}T = C_3 \tag{24}$$

Bearing in mind that the cutting speed for 60 minutes tool life is 100 m/min, eqs. (22), (23), (24) yield the equation:

$$vs^{0.36}a^{0.093}T^{0.315} = 242 \tag{25}$$

The exponent of T decreases with increasing A, while the constant on the right-hand side of eq. (25) increases with decreasing K_D.

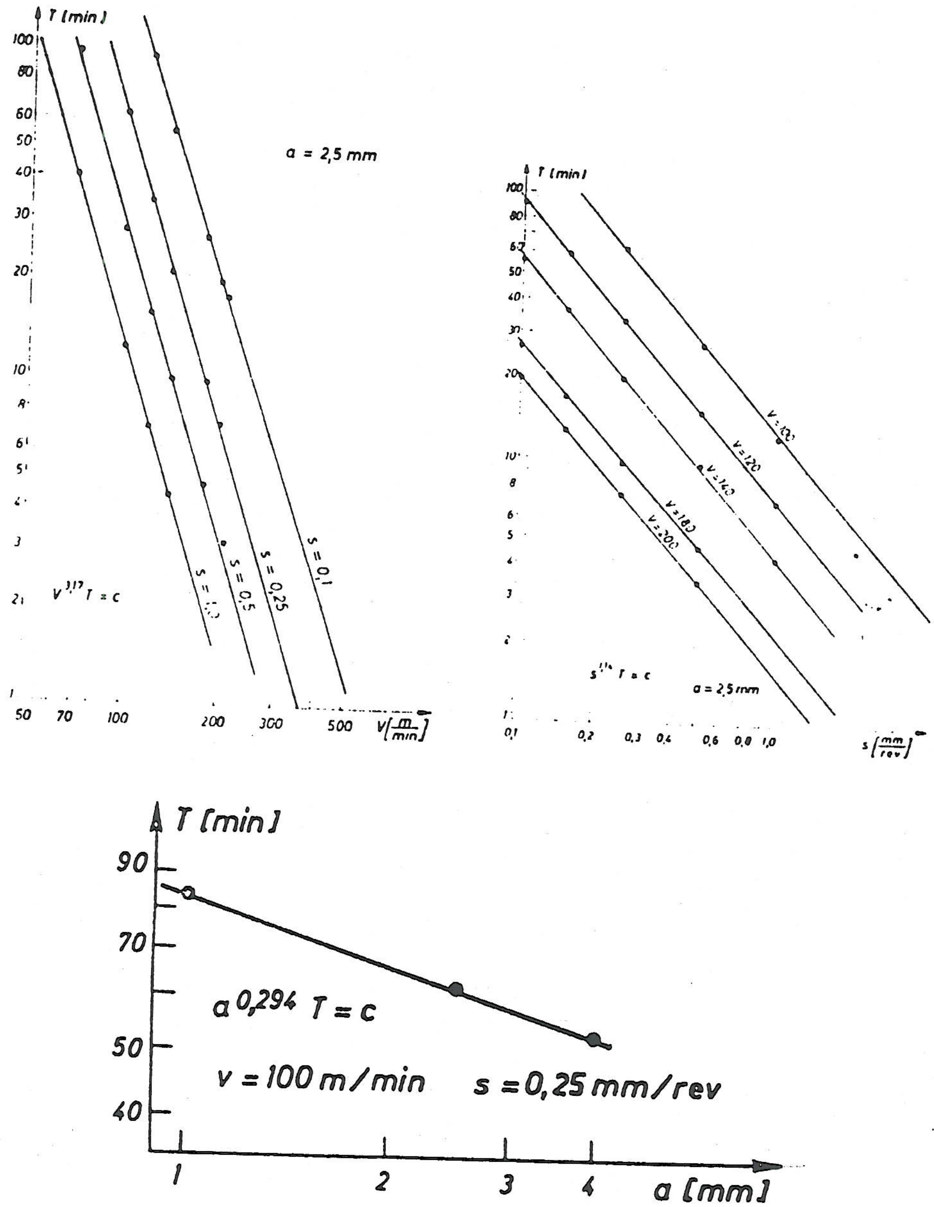

Figure 6. Tool life V. cutting speed (a), feed (b), depth of cut (c)

PARTIAL LINEARIZATION OF MODEL

The section deals with linearization of exponent:

$$\exp\left[-A/(273+\theta)\right] = M + N\theta \tag{26}$$

where

$$M = \left[1 - A\theta_o/(273+\theta_o)^2\right] \exp\left[-A/(273+\theta_o)\right] \tag{27}$$

$$N = \left[A/(273+\theta_o)^2\right] \exp\left[-A/(273+\theta_o)\right] \tag{28}$$

The linear model is shown in Fig. 7, where

$$F'_o = F_o + m\Delta F \tag{29}$$

where $\Delta F = B_o C_w a$ and $m < 0.5$.

$m\Delta F$ supplements force F_o, as the latter increases during the cutting process, and should be brought to an average working point on the exponential curve.

Defining

$$\begin{aligned}
K_1 &= K_T v^{0.95} s^{-0.55} a^{-0.95} N K_D \\
K_2 &= K_T v^{0.95} s^{-0.55} a^{-0.95} M K_D / \theta_o \\
K'_F &= K_F/(s \cdot a) \\
C'_w &= C_w a \\
\tau &= \tau_n, \ (\tau_n \to 0),
\end{aligned}$$

we obtain, after Laplace-transformation

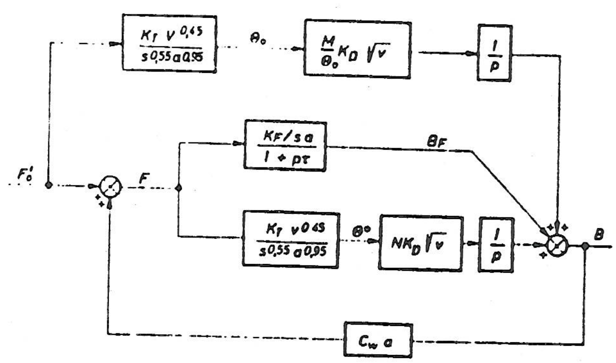

Figure 7. Block diagram of linear model

$$B(p) = \frac{p(K'_F + K_1\tau + K_2\tau) + (K_1 + K_2)}{p^2\tau + p[1 - (K'_F + K_1\tau)C'_w] - K_1 C'_w} F(p) \tag{30}$$

since F_o is constant, we have:

$$F(p) = \frac{F'_o}{p} \tag{31}$$

and finally, defining

$$\begin{aligned}
b &= (K'_F + K_1\tau) C'_w \\
d &= [(1 - b)^2 + 4K_1 C'_w \tau]^{\frac{1}{2}} \\
e &= K_2/K_1 \\
\frac{1}{\tau_1} &= \frac{1}{2\tau}(d + b - 1) \\
\frac{1}{\tau_2} &= \frac{1}{2\tau}(d - b + 1) \\
D &= \frac{1}{2d}[1 + b - d - e(-1 + b + d - 2K_1 C'_w \tau)] \\
C &= \frac{1}{2d}[1 + b + d + e(1 - b + d + 2K_1 C'_w \tau)]
\end{aligned}$$

we obtain after inverse Laplace transformation:

$$B(t) = \frac{F'_u}{C'_w}\left[-(1+e) + C\exp(t/\tau_1) - D\exp(-t/\tau_2)\right] \tag{32}$$

it being obvious that $D > 0$ (see Appendix II).

A numerical calculation shows that for $v = 100$ m/min and $s = 0.25$ mm/rev τ_1 is about 70 min, and τ_2 about 5 min.

It is seen that the wear diagram in the first and second stages comprises three curves, the interval taken as linear being in fact exponential with a relatively large time constant. It is also seen that assumption of $t \ll \tau_1$ in eq. (32) yields eq. (3), shown by Müller to be good approximation to a practical curve. (From eq. (32): for $t = 0$, $B = 0$, so: $D = C - (1 + e)$.) In addition, two practical diagrams were taken from Ref. [5] and mathematical equations of the type of eq. (32) fitted to them (see Fig. 8).

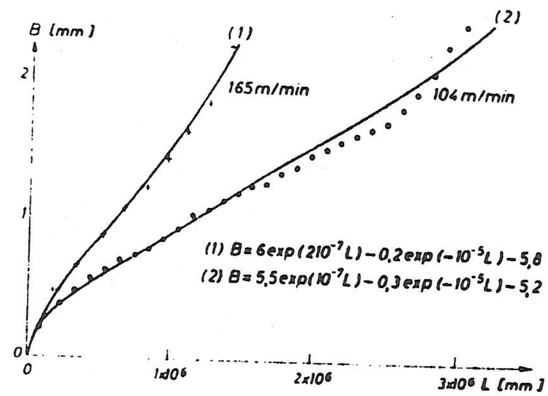

Figura 8. Measured and calculated wear curves
CX(AA) carbide; AISI 4340 steel
s = 0,0012 ipr; a = 0,5 mm

A linear model with the numerical values given in the preceding section and $m = 0.17$, was simulated on a computer. Table I lists some constants for eq. (32). (The constants are multiplied by F'_u/C'_w.)

Table I. — Results of linear model simulation

No.	v[m/min]	s[mm/rev]	$(1+e)$	C	D	$1/\tau_1$	$1/\tau_2$
1	100	0.1	0.110	0.264	0.154	0.006	0.171
2	100	0.25	0.274	0.395	0.120	0.016	0.189
3	100	0.5	0.525	0.625	0.096	0.029	0.195
4	180	0.5	0.602	0.672	0.070	0.183	0.353

In all cases $a = 2.5$ mm.

Table I and Fig. 8 show that the constants vary as follows with the speed:

1. $1/\tau_1$ increases as the power in Taylor s equation, $v^n T = C_1$.
2. $1/\tau_2$ increases linearly.
3. D decreases, while C and $(1 + e)$ increase.

CONCLUSIONS

A mathematical model was developed for steel turning with a carbide tool and verified on practical wear-time curves taken with the cutting speed as parameter.

The model also yielded the tool life as function of the cutting speed, feed and depth of cut, which in turn lead to Taylor's equation.

Linearization yielded results close to those of the original model, and provided a formula for the time behavior of the tool wear. This formula permits prediction of tool life in the course of optimization of the cutting process.

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APPENDIX I

DERIVATION OF DIFFUSION EQUATION

The equation of transport by molecular diffusion between two materials at constant density is:

$$\frac{\partial \rho}{\partial t} + \rho \nabla \cdot v + \nabla \rho \cdot v = D \nabla^2 \rho + r \tag{I-1}$$

where

D = diffusion coefficient,
ρ = mass density,
v = mass velocity,
r = rate of inner production.