The Optimal Locus Approach With Machining Applications

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An optimal locus concept is introduced as the basis for an optimization methodology for real-time control subject to time-varying constraints. The optimal locus in the control plane contains all possible optimum points, and the actual point is found at the intersection of the optimal locus with the most limiting constraint. The mathematical basis of the approach is a given set of equations which is less than the number of unknowns, and the addition of real-time measurements to compensate for the missing information. The control system generates the optimal parameters in real time, and uses them as references to the control loops. The optimization methodology and control architecture are structures in a generalized way for application to processes having multiple variables and subject to several constraints. The proposed controller architecture can effectively control many machining processes. The optimal locus approach was applied to a grinding system and the experimental results verify the proposed theory.

1 The Evolution of Control Systems

World War II was a milestone in the development of control theory. The major theme of this new discipline in the forties and fifties was the concept of *feedback* to assist in producing desirable responses of processes. In the early sixties, the *optimal control* theory was developed. By including the required specifications in a performance index, optimal control was used to solve the control problems by applying mathematical methods to optimize a performance index [1-4].

Many control systems suffer from inadequate performance because of inaccurate modeling of the controlled process or because it changes outside the range of compensation of a conventional feedback controller. To remedy this situation, control research in the late sixties and early seventies was directed toward the development of methods for on-line *estimation* and *identification* of processes [5-7]. The main idea behind these methods is the on-line improvement of the knowledge of the model of the process. Some attempts have been done to apply this theory to cutting processes [8, 9], but it has never been applied to practical systems. In parallel to the research in process identification, *adaptive control* was developed to treat situations where uncertainties in the process are a factor in the design of the controller [10, 11]. Adaptive control was also applied to machining processes [12-14], and when combined with model estimation techniques demonstrated satisfactory results in research laboratories [15, 16].

Adaptive control and on-line identification can be considered as the first step toward self-learning [17, 18] and intelligent control systems. *Learning systems* have been defined to represent processes in which dynamic accumulation of information takes place in order to improve the understanding of the process and make decisions [18-20]. *Intelligent control* would replace the human judgment in making decisions (e.g., type and level of control variables), planning control strategies, and learning new functions by training other intelligent functions [21].

The control of machining processes has not followed the same evolutionary stages of general control systems. Although real-time estimation techniques and sophisticated adaptive control systems were demonstrated in research laboratories for milling and turning, they have not been applied in practice. Simple feedback loops are used in most CNC systems. Even in the most advanced CNC systems, the part programmer still determines the machining parameters, such as feed and cutting speed. An improved system would be one in which the optimal values of these parameters are generated in real time by the controller according to the behavior of the machining process, and subsequently used as velocity references in the control loops. Once the system is initialized, at any arbitrary operating point, it should automatically converge toward the best operating point which guarantees optimal control of the process. Such a controller is defined here as an *intelligent machining controller* (IMC) since it replaces the human judgment in making optimal machining decisions. The purpose of this paper is to introduce the concept of the optimal locus which is the basis of the IMC architecture.

2 Conventional Calculation of an Optimal Machining Point

From the literature dealing with economics of machining processes (e.g., [22]), a cost function is usually formulated as:

$$\phi_1 = C_1 t + (C_2 + C_1 t_1)t/T \tag{1}$$

where

ϕ_1 = cost to remove volume V needed to produce a part ($)

C_1 = cost of machine and operator per unit time ($/min)

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C_2 = cost per cutting edge or regrind per change ($)

t = machining time per part (min)

t_1 = tool changing time (min)

T = tool life (min)

For rectilinear turning, for example, $t = \pi DL/vf$ where D and L are the workpiece diameter and length, v is the cutting speed and f is the feed. A general definition of t is the time needed for removing a volume of material for one part, V, in which case ϕ_1 is determined by $t = V/\dot{V}$ (in turning $\dot{V} = vfa$, where a is the depth of cut). By dividing ϕ_1 by a constant value, (e.g., πDLC_1 in turning), a more convenient cost function is obtained

$$\phi = \frac{1}{vf}\left(1 + \frac{\tau}{T}\right) \tag{2}$$

where τ is a constant

$$\tau = t_1 + C_2/C_1 \tag{3}$$

Optimizing the process for minimum cost generally requires finding the operating parameters (e.g., v and f in turning) which will minimize ϕ subject to constraints on their allowable values.

If the optimization criterion is the minimum production time rather than cost per unit, the same approach is used but with $\tau = t_1$ in equation (2), namely, the cost of the tool C_2 is neglected in this case.

In milling and turning, a maximum feed (f_x) constraint is usually active, namely,

$$f \leq f_x \tag{4}$$

The constraint on the maximum feed may be dictated by such factors as the required surface finish or the limiting cutting force.

With the conventional economic analysis, $f = f_x$ and the Taylor's tool life equation

$$Tv^n = C_0; \quad n > 1 \tag{5}$$

are substituted into equation (2), and the optimal cutting speed v_0 is obtained by setting the first derivative of ϕ with respect to v equal to zero. This yields

$$v_0 = \left[\frac{C_0}{\tau(n-1)}\right]1/n \tag{6}$$

However, equation (5) is only an approximate representation of data points, for limited values of v and f, and therefore the obtained v may not be very accurate especially if the permissible operating region of v and f is large.

3 The Optimal Locus Concept

From the optimal control viewpoint, many machining optimization problems may be characterized as follows:

1 A performance index in a form of a cost function (e.g., equation (1)).

2 Two control variables (a control variable is identical to a machining parameter, and can be the feedrate, the cutting speed, etc.).

3 Fixed constraints on the control variables (e.g., equation (4)).

4 Constraint(s) depending on both control variables, as well as on a state variable which varies with machining time and is inaccessible to in-process measurement (e.g., tool wear). Such a constraint is defined as a *varying constraint*.

5 A measurable output variable(s) which can indicate a violation of the varying constraint(s).

Examples of varying constraints are the maximum allowable machine power in milling (depends on the cutting speed), the force which breaks the tool in turning (depends on the feed and wear), or the power causing workpiece burn (thermal damage) in grinding. The corresponding measurable output variables are the machine power, the cutting force, and the grinding power, respectively. In this section the general analytical approach of the optimal locus is introduced, while the next section shows how the optimal locus approach can be applied to machining.

The optimization problem with one varying constraint can be stated as follows: Minimize (or maximize) the cost function $\phi(u, v)$ subject to the varying constraint $P(u, v, x) \leq 0$, where u and v are the control variables and x is an inaccessible state variable.

The solution might be divided into two cases:

A. The optimal control point is not on the constraint, namely $P < 0$. The solution is obtained by solving the equations

$$\frac{\partial \phi}{\partial u} = 0; \quad \frac{\partial \phi}{\partial v} = 0 \tag{7}$$

The solution of this case is straightforward, but in machining applications, it rarely occurs. If only one fixed constraint is active (e.g., equation (4)), the solution is given by one of the equations in (7).

B. The optimal point is on the varying constraint, which, in turn, becomes an equality constraint $P = 0$. In this case the optimization problem can be solved by using the Lagrange method as follows. First we define the Lagrangian

$$L(u,v,x) = \phi(u,v) + \lambda P(u,v,x) \tag{8}$$

where λ is the Lagrange multiplier. We now adjust u and v such that L is a maximum (or minimum) for each particular value of x. This requires that

$$\frac{\partial L}{\partial u} = \frac{\partial \phi}{\partial u} + \lambda \frac{\partial P}{\partial u} = 0 \tag{9}$$

$$\frac{\partial L}{\partial v} = \frac{\partial \phi}{\partial v} + \lambda \frac{\partial P}{\partial v} = 0 \tag{10}$$

and

$$\frac{\partial L}{\partial \lambda} = P(u,v,x) = 0 \tag{11}$$

Eliminating λ by combining equations (9) and (10) yields

$$\frac{\partial \phi}{\partial u}\frac{\partial P}{\partial v} - \frac{\partial \phi}{\partial v}\frac{\partial P}{\partial u} = Q(u,v,x) = 0 \tag{12}$$

For any particular x, the solution of equations (11) and (12) provides the optimal point u_0, v_0. The collection of all these points in the uv-plane is the *optimal locus*. Its equation

$$l(u,v) = 0 \tag{13}$$

obtained by substituting x from equation (11) into (12), is independent of the inaccessible state x! This fundamental optimization equation of the controlled process, is the basis of the Intelligent Machining Controller. The solution for an optimal point is obtained analytically by solving equations (11) and (13), and graphically by finding the intersection between the constraint $P(u, v, x) = 0$ and the optimal locus $l(u, v) = 0$.

The following example, although having no physical meaning, clarifies the analytical approach of the optimal locus.

Example: Minimize the cost function

$$\phi = (u-1)^2 + (v-4)^2 \tag{14}$$

subject to the fixed constraint

$$5 > u > 0 \tag{15}$$

and the varying constraint

$$P = xv - u - 1 \leq 0 \tag{16}$$

where x varies in the range $20 \geq x \geq 0.2$.

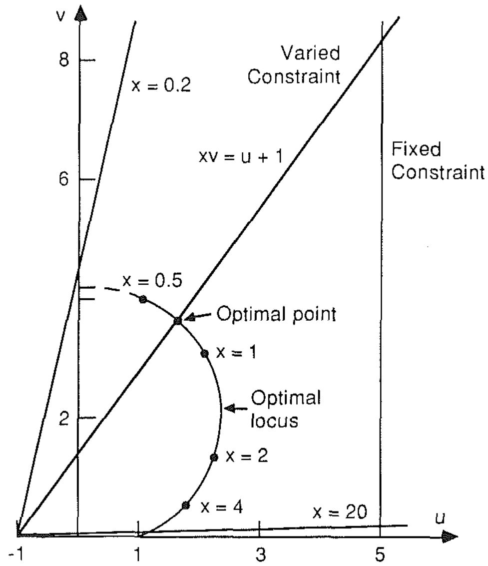

Fig. 1 An example of a calibrated optimal locus

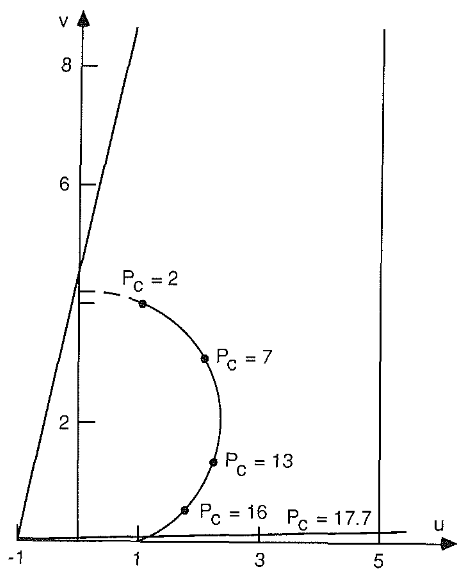

Fig. 2 Optimal locus calibrated in P_C units

A. First the unconstrained problem is solved. According to equation (7) the solution is $u_0 = 1$, $v_0 = 4$. Substituting these values into (16) yields $x \leq 0.5$, which means that in the range $0.5 \geq x \geq 0.2$ the varying constraint (16) is not active, namely, $xv_0 - u_0 - 1 < 0$. Therefore, for x in this range the optimal operating point is (1,4).

B. For an active varying constraint, equation (12) in this example becomes

$$(u-1)x + (v-4) = 0 \tag{17}$$

Eliminating x by combining equation (17) and the equality of (16) yields the equation of the optimal locus (i.e., equation (13))

$$u^2 + (v-2)^2 = 5 \tag{18}$$

The optimal locus and the constraints for this example are shown in Fig. 1. The optimal locus is calibrated in x values. For $x < 0.5$, the constraint is not active and the optimal point is fixed at (1,4). For $x > 0.5$ the optimal point is at the intersection of the constraint and the optimal locus.

In many practical cases the varying constraints are composed of two components

$$P(u,v,x) = P_c(u,v,x) - P_b(u,v) \leq 0 \tag{19}$$

where P_c is the dynamic component of the constraint which is affected by an inaccessible state variable, and P_b is a *corresponding measured variable*. The strength of the optimal locus approach is in cases where the inaccessible state variable strongly varies as the process progresses.

For example, let us assume that the loading torque on a DC motor, which drives a robot joint, gradually varies because of the changing inertia

$$T = [J_0 + x(t)]\mathring{\omega} \tag{20}$$

where x represents an inaccessible variable which depends on the motion of the other axes. The motor speed ω, which also varies, is measured with a tachometer. The maximum torque (T_m) and speed of the motor are limited. The maximum power that can be delivered by the motor is

$$P_b = \omega T_m \tag{21}$$

and since ω is measured, P_b can be continuously calculated. The actual power (the corresponding variable here) is

$$P_c = \omega T(x) \tag{22}$$

where T is given in equation (20). The power P_c is measurable in real time, and the varying constraint is $P_c \leq P_b$ or $P_c - P_b \leq 0$.

To obtain the optimal locus, the variable P in equations (11) and (12) may be substituted by P_b and P_c from equation (19), which yields

$$P_c(u,v,x) = P_b(u,v) \tag{23}$$

and

$$\frac{\partial \phi}{\partial u}\left(\frac{\partial P_b}{\partial v} - \frac{\partial P_c}{\partial v}\right) = \frac{\partial \phi}{\partial v}\left(\frac{\partial P_b}{\partial u} - \frac{\partial P_c}{\partial u}\right) \tag{24}$$

The optimal locus equation $l(u, v) = 0$ is obtained as before by eliminating x from equations (23) and (24). Conceptually, the pair equations (11) and (12) is equivalent to equations (23) and (24). The latter, however, lays the ground for the implementation of a real-time optimal controller.

The strength of the optimal locus approach is in cases where P_c can be measured in real time. Since the intersection between $P_c = P_b$ and $l(u,v) = 0$ provides the optimal point, the locus can be calibrated in P_c units and subsequently stored in the control computer. The actual value of P_c is then used to retrieve the optimal point in real time. Variations in the state variable x are sensed indirectly through the measurements of P_c, and the resultant optimal point is automatically adapted to the process variations. Conceptually, the optimal locus approach is a general method which compensates for the absence of knowledge on a process state x by measuring a corresponding variable P_c that is a part of a related constraint.

To further explain the method let us assume in the above example that

$$P_b = 17 + u - 4v \tag{25}$$

and

$$P_c = xv + 16 - 4v \tag{26}$$

Substituting values of x and v that are on the locus into equation (26) permits its calibration in P_c units, as shown in Fig. 2. Equation (18) with the P_c calibration is stored in the computer. A subsequent measurement of the value of P_c in real time enables the automatic retrieval of the optimal points u and v.

If the model in equation (26) has an uncertainty of the form

$$P_c = mxv + 16 - 4v$$

where m varies in the neighborhood of $m = 1$, the optimal locus given by equation (18) is still correct for this example, and only its calibration changes. However, if P_c is monitored in real time, the exact optimal point will be retrieved despite the variations in the model. This is, however, a very favorable example. In the general case modeling uncertainties might cause operation in the neighborhood of the optimal point, but still the optimal locus approach always guarantees better results than off-line optimization methods.

The optimal locus approach can be extended to deal with more than two control variables, and also to cases in which ϕ is a function of x. However, these cases are not typical in machining and will complicate the architecture of the machining controller which implements the proposed optimization method, and therefore are not discussed here.

4 The Optimal Locus in Machining

Varying constraints of the type

$$P = P_b(v,f) - P_c(v,f,x) \geq 0$$

are frequently imposed in machining. Here v and f are defined as two general machining parameters, P_b is a physical limit, and P_c is its corresponding measurable variable. For example, if f is the feed in turning, P_b can be the tool breakage force

$$P_b = Kf^e \tag{27}$$

and then P_c is the measured cutting force; or P_b can be the maximum power of the machine

$$P_b = Kvf^q \tag{28}$$

where f is the feed and v is the cutting speed, and then P_c is the actual machine power; or P_b can be the burning power in grinding [23]

$$P_b = K_1 f + K_2(vf)^l \tag{29}$$

with v being the speed and f the radial infeed, and then P_c is the actual net grinding power.

In order to apply the concept of the optimal locus, the corresponding variable must be known or measured in real time. In the general case the corresponding variable depends on f and v, and a state variable. For example, in turning [24]

$$P_c = Kf^q - C_v v + C_w W \tag{30}$$

where W, the width of the flank wear, is the inaccessible state variable, and K, q, C_v, and C_w are constants.

To solve the optimization problem with the varying constraint, the Lagrange multipliers method with the equality constraint $P = P_b - P_c = 0$ is applied. Using a Lagrange multiplier λ, the optimization problem becomes one of extremizing the function

$$L = \phi + \lambda(P_b - P_c) \tag{31}$$

where ϕ is given in equation (2). As in equations (9) to (11), the necessary condition is for the partial derivatives of L with respect to v, f, and λ to vanish, which leads to

$$1 + \frac{\tau}{T}\left[1 + \frac{\partial(\log T)}{\partial(\log v)} = \lambda v^2 f\left(\frac{\partial P_b}{\partial v} - \frac{\partial P_c}{\partial v}\right)\right. \tag{32}$$

$$1 + \frac{\tau}{T}\left[1 + \frac{\partial(\log T)}{\partial(\log v)} = \lambda f^2 v\left(\frac{\partial P_b}{\partial f} - \frac{\partial P_c}{\partial f}\right)\right. \tag{33}$$

and

$$P_b = P_c(x) \tag{34}$$

Eliminating λ by combining equations (32) and (33) yields the particular case of equation (12) for milling, turning, and drilling:

$$\left[1 + \frac{\tau}{T}\left(1 + \frac{\partial \log T}{\partial \log v}\right)\right]\left(\frac{\partial P_b}{\partial f} - \frac{\partial P_c}{\partial f}\right)f = \left[1 + \frac{\tau}{T}\left(1 + \frac{\partial \log T}{\partial \log f}\right)\right]\left(\frac{\partial P_b}{\partial v} - \frac{\partial P_c}{\partial v}\right)v \tag{35}$$

The optimal locus equation is obtained by substituting x from equation (34) into (35). However, for particular cases in which $\partial P_c/\partial v$ and $\partial P_c/\partial f$ are independent of x (see example below), equation (35) is the optimal locus equation. The optimal operating point is determined either analytically by solving equations (34) and (35) (or equation (34) and the locus equation) for f and v, or graphically by finding the intersection of the optimal locus with the constraint (34).

The optimal locus approach establishes a science-base to intelligent machining and new design approach of machine tool controllers. This statement is supported by the following:

1 The approach is general and fits a variety of different processes which might be subject to constraints of different types. The strength of the approach is the independence of the optimal locus equation from unmeasurable state variables. The effect of these variables on the optimal point is inserted through measured variables related to the constraints.

2 No specific tool life equation is assumed and therefore any type of tool failure can be considered. It might be thermal damage in grinding [23]; or tool wear or breakage in milling, turning, and drilling, (e.g., [25]); or it might be attributed to both wear and breakage [26-29]. Since no single mode of tool failure can account for even the majority of failures [30, 31], introducing an approach which mathematically accommodates various failures is significant.

3 Two distinct modes of tool failure might be mathematically accommodated, one by the cost equation and the other by the constraint. For example, the tool life in the cost equation can be defined by the width of the flank wear, and the varying constraint will protect the tool from breakage.

4 A constraint might be given as a function of the machining parameters and a state variable, thereby becoming a varying constraint, which varies as the process proceeds. For example, the power in grinding increases as the grinding wheel becomes dull (large effective wear flat area) [23, 32]. With the proposed approach this can be automatically compensated by a shift on the optimal locus and operating with smaller workpiece speed and infeed velocity [33].

5 Although the approach was demonstrated with one varying constraint, it can be expanded to any number of constraints, a case in which λ in equation (8) becomes a vector rather than scalar. An example with two varying constraints (burning power and surface finish) can be found in [33].

6 A variety of optimization criteria might be set, depending on the definition of the constant τ: Minimum cost per part (equation (1)); maximum production rate ($\tau = t_1$), or a value ($t_1 < \tau < t_1 + C_2/C_1$) for maximum profit [34].

7 The optimal locus approach assumes variables which can be measured in practice (e.g., power or cutting force), and therein lies the grounds for real-time intelligent controller.

5 The Intelligent Controller Architecture

The architecture of the intelligent machine controller (IMC) is based upon the optimal locus theory which was previously introduced, and is presented in Fig. 3. The use of the term "intelligent" is justified if a controller replaces the human judgment in making decisions [21], which happens at the IMC regarding the determination of the optimal machining variables. The "optimal locus algorithm" in Fig. 3 is based on the optimal locus equation, and the "calculated constraint" is the P_b equation. The varying constraint is represented by three parts: the calculated P_b, the measured P_c, and the software comparator. The system starts to operate at an arbitrary small f, and the counterpart v is determined in real time from the optimal locus equation. The pair v and f are transmitted as references to the control loops, and also used to calculate the P_b portion of the constraint. The new P_b is compared with the measured P_c, and the error e is used to generate a corrected value f (with the aid of a PID, or other controller). The system quickly converges along the optimal locus curve toward the

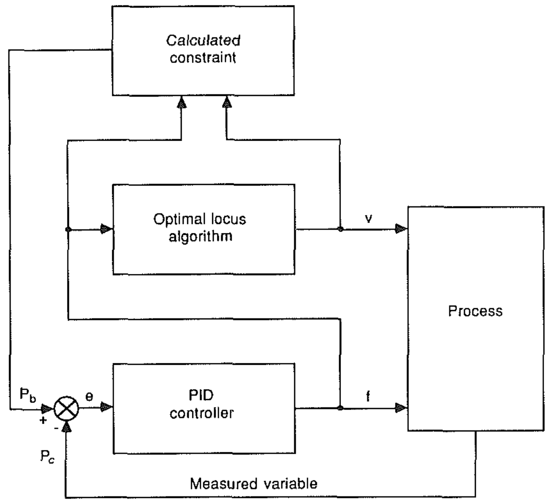

Fig. 3 The intelligent machining controller (IMC)

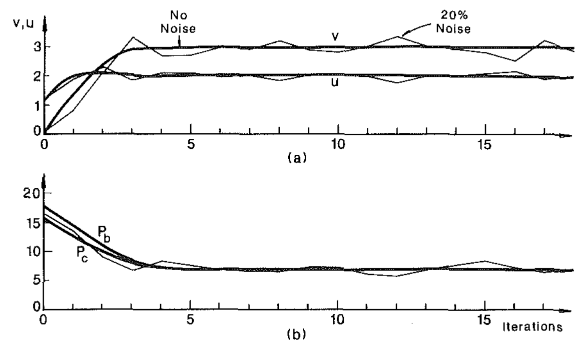

Fig. 4 Convergence to an optimal point with no-noise and 20 percent measurement noise. (*a*) The control variables, (*b*) the constant P_b and the measurement P_c.

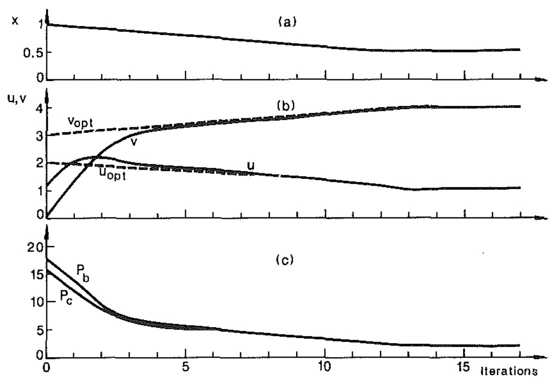

Fig. 5 Convergence to a time-varying optimal point. (*a*) The varying parameter, (*b*) the control variables, (*c*) the constraint and the measurement.

optimal point. The integral component in the PID controller guarantees that at steady state $P_b = P_c$, namely equation (34) is satisfied. If during machining P_c varies (e.g., because of tool wear), the operating point moves and the calculated P_b is shifted as well; the system will stabilize at a new point on the optimal locus.

From control theory viewpoint the IMC has an interesting structure: The references to the control loops are not supplied by external sources. All the references (i.e., v, f, and P_b) are internally generated, namely, the system possesses enough "intelligence" to know the desired values of the references and to modify them as the cutting progresses. Such a system is being defined as an intelligent machining system.

Example. The proposed control architecture has been simulated on the process given in Section 3. The optimal locus, the constraint portion P_b, and the measurement P_c are given in equations (18), (25), and (26), respectively.

First the designer must select an input variable(s) to the locus algorithm. In this example each v has a unique positive u counterpart, but u might have two positive solutions for v. Therefore, v is selected as the input variable to the locus algorithm, and the counterpart u is calculated from equation (18). Notice that equation (18) is valid only at a limited region, as shown in Fig. 1, and the locus is not a full circle. Next, a loop controller algorithm should be selected. It must include an integral action, since at the steady state the objective is to obtain $P_b = P_c$. The algorithm in this example is

$$v(i+1) = v(i) + k(P_b - P_c)$$

where $k = 0.6$. This value was found by trial-and-error; larger values (e.g., $k = 0.9$) provide larger errors of the parameters at the steady state; smaller values (e.g., $k = 0.2$) increase substantially the number of iterations without significant improvement in the accuracy.

The next step is to decide upon an initial operating point. To eliminate constraint violations this point must guarantee $P_b > P_c$ for any value of x. In this example, at the intersection point of the lower constraint with the locus ($v = 0.109$, $u = 1.194$) the condition $P_b > P_c$ is always satisfied (see equations (25) and (26)), and therefore it is selected as the starting point.

In the first simulation we assume that the process state varies very slowly compared with the convergence rate of the optimal locus algorithm, and therefore x is a constant during the transient period. The process was simulated with $x = 1$; the analytic solution is $v = 3$, $u = 2$, and $P_b = P_c = 7$. As shown by the thick lines in Fig. 4, after three iterations the errors in u, v, and P_b are less than 4 percent, and after five iterations less than 0.5 percent (compared with the analytic solution). At each iteration the operating point is on the locus, and therefore the condition $P_b > P_c$ is satisfied along the convergence trajectory. That means that the optimal locus approach also provides a convergence strategy that eliminates constraint violations.

When 5 percent random noise (a practical number) is added to the measurement P_c, the resultant errors are so small that they can not be depicted graphically (the graphs actually coincide with the thick no-noise lines). The results of adding 20 percent noise are shown in thin lines in Fig. 4 (P_b is not shown for this case). The process continues to operate in the neighborhood of the optimal point, and the measured constraint violations (not depicted) are smaller than 2 percent. (In actual system violations do not occur since the noise is filtered.)

The simulation results of a time-varying process are shown in Fig. 5. Here, the value of x is gradually changed from $x = 1$ to $x = 0.52$. The control variables u and v track the analytic optimal solution with error smaller than 3 percent (after 4 iterations), and the constraint is never violated. Constraint violations are eliminated since the convergence trajectory is the optimal locus itself, as shown in Fig. 6 for the first 14 iterations.

6 The IMC in Grinding

The optimal locus approach was applied to an operating experimental grinding system [35]. The two controlled grinding parameters are the surface speed v_w (or alternatively the workpiece spindle speed n_w) and the radial infeed f (or alternatively the infeed velocity v), and the varying constraint is the burning power P_b. In theoretical analyses the pair (v_w, f) is usually applied, but in practical systems n_w and v are the ac-

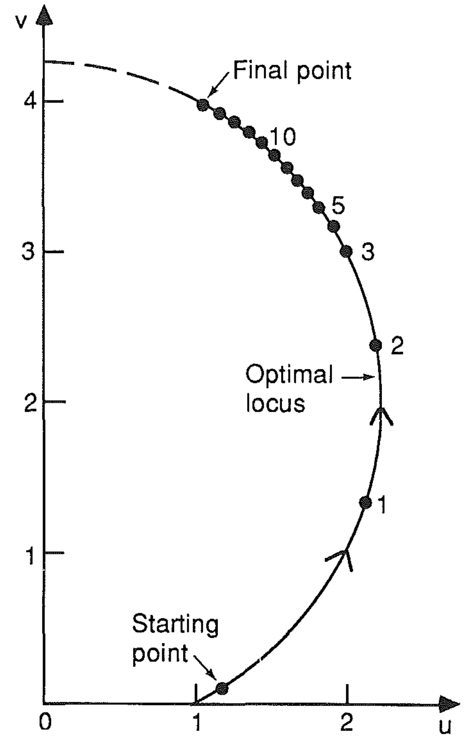

Fig. 6 Convergence trajectory along the optimal locus

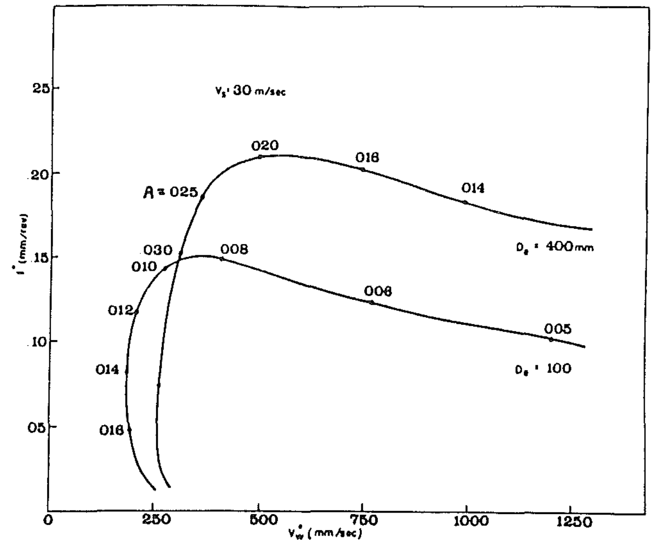

Fig. 7 Optimal loci in grinding, calibrated by the wheel's wear flat area (A); the grinding-wheel equivalent-diameter (De) is a parameter (constant peripheral wheel velocity V_S = 30 m/s)

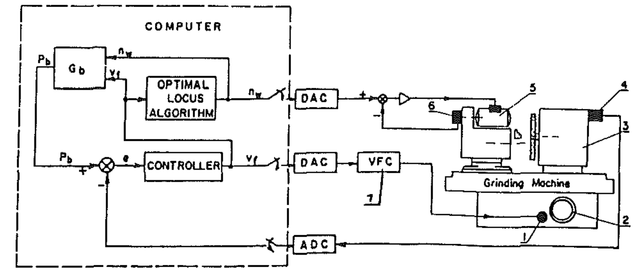

Fig. 8 Intelligent controller for a grinding machine: 1-stepping motor infeed drive; 2-infeed control handwheel; 3-grinding wheel motor; 4-power sensor; 5-workpiece spindle DC motor; 6-tacho-generator; 7-voltage-to-frequency converter

tual control variables. Based on the approach presented here, optimal loci for grinding were obtained (see Fig. 7) and stored in the computer. Measurements of the grinding power P_c are fed from the grinding machine to the computer, and the computer, in turn, controls the parameters v and n_w to operate along the optimal locus and converge toward the optimal working point.

The experimental intelligent grinding system is shown in Fig. 8 [35]. The workpiece spindle is driven by a dc servomotor in the continuous range 0.4 to 63 r/s. For controlling the radial infeed velocity v, a stepping motor drive was attached to the infeed control handwheel of the machine. A Hall element sensor was connected to the main drive motor to measure the machine power. During the grinding operation, the on-line measured power is fed into the computer through an ADC, and the net grinding power P_c is obtained by subtracting the idling power from the measured power.

The control system is of the sampled-date type with sampling period of 0.5 s. At constant time periods the measured power P_c is sampled; the burning power P_b is calculated in real time using the previous v and n_w values. The new control parameters v and n_w are then assigned and transmitted (through DACs) as references to the control loops. Between sampling events, the values of the control parameters are kept constant by storing their values in computer registers assigned to the DACs.

The heart of the control system is an algorithm incorporating the optimal locus equation described in the previous section. The grinding operation might start at an arbitrary point in the vn_w-plane, but is immediately transferred by the computer to a point on the optimal locus by changing n_w. Thereafter, the trajectory of convergence toward the optimal operating point is along the optimal locus. For controlling the convergence, instead of providing an external reference as is typically done in control loops, the reference to the control loop, P_b, is calculated in the block G_b to which the control variables v and n_w are fed. As a consequence, the convergence rate depends on the error $e = P_b - P_c$, which gradually converges to zero when proceeding along the locus.

From preliminary grinding experiments, the grinding power P_c was found to be much more sensitive to the infeed velocity v than to the spindle speed n_w. Therefore, only v is directly determined by the controller in Fig. 8 and the corresponding n_w is calculated on the optimal locus. Such a single-input-single-output controller is much simpler to design than a multioutput controller. The controller algorithm was selected according to the equation:

$$v(i) = v(i-1) + K_1 e(i) + K_2[e(i) - e(i-1)] \quad (36)$$

where the index i is the number of the sampling event. The first two terms on the right-hand side of equation (36) constitute an integral controller of gain K_1 which ensures a zero error ($e = 0$) at the optimal point. The last term is essentially a derivative controller of gain K_2 which was added to decrease the tendency for overshooting while converging toward the optimal point. The controller gains of the pilot system were selected as $K_1 = 3.3\ \mu\text{m}/(\text{s}\cdot\text{kW})$ and $K_2 = 5.3\ \mu\text{m}/(\text{s}\cdot\text{kW})$.

For repetitive grinding of identical parts (referred to as cycles), the system makes use of a learning starting point. After the first part is ground, the system selects the optimal grinding conditions obtained with that part as a starting point for the next part. The stepping motor runs at a constant high acceleration to this starting point, and then the control is switched to the IMC mode.

Some results illustrating operation of the grinding system for repetitive grinding cycles, with only the grinding power constraint, are presented in Fig. 9. For cycle 1, it can be seen when starting from an initial infeed velocity $v = 1\ \mu$m/s that it

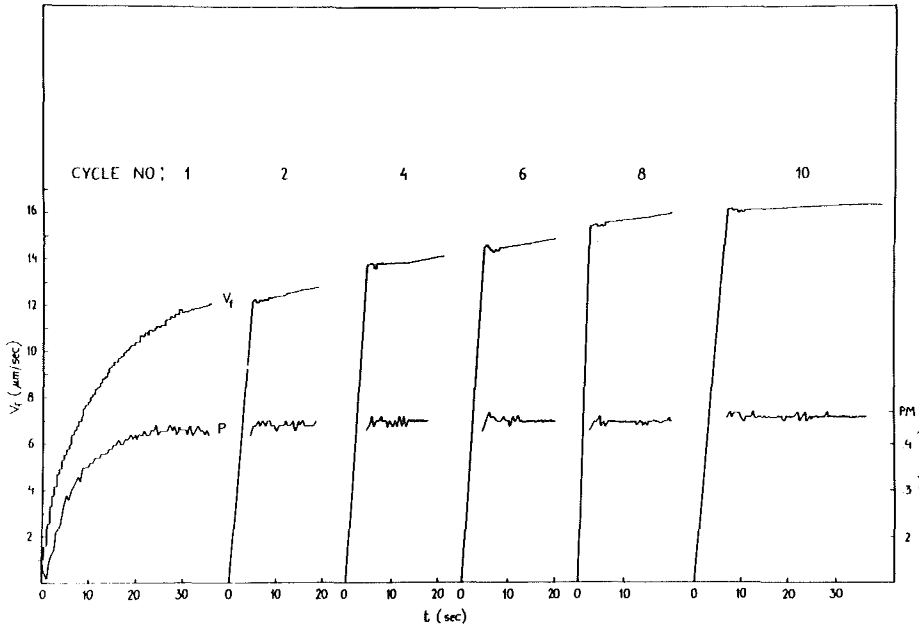

Fig. 9 Convergence to a varying optimal point in grinding by using learning starting point on the optimal locus (conditions: 32A46K8VBE wheel, SAE4340 steel workpiece, 0.3 mm removed from workpiece radius per cycle)

takes 28 s for the process to converge near to the optimal point. In the subsequent cycles, the system accelerated to the learning starting point, and the convergence time was reduced to about 4 s.

7 Conclusions

The paper introduces a novel on-line optimization approach, based on the determination of an optimal locus for processes subject to time-varying constraints. The constraints depend on unmeasurable state variables. The optimal locus equation, however, is independent of these unmeasurable state variables, and they only affect the location of the actual optimal point on the locus. The effect of the unknown state variables is inserted through real-time measurements that are contained in the constraint. The optimal operating point lies at the intersection between the optimal locus and the most limiting constraint, and can be determined in real time. The methodology also uses the optimal locus as the convergence trajectory to the optimal point, and thereby guarantees elimination of constraint violations during the transient periods.

The optimal locus approach has established a science base to the development of intelligent machining controllers that automatically calculate the machining variables and adapt them to the process and its constraints. The IMC was applied to a grinding machine, and the experimental results verify the theoretical analysis. The system always converged to the optimal point. It is expected that the optimal locus approach and the associated IMC will be applied in future generations of machine tool controllers.

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