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MODELING OF MANUFACTURING COMPLEXITY IN MIXED-MODEL ASSEMBLY LINES

Xiaowei Zhu , S. Jack Hu, and Yoram Koren
Department of Mechanical Engineering
University of Michigan
Ann Arbor, MI 48109, USA

Samuel P. Marin
Manufacturing Systems Research Lab
General Motors R & D Center
Warren, MI 48090, USA

ABSTRACT

Mixed-model assembly lines have been recognized as a major enabler to handle product variety. However, the assembly process becomes very complex when the number of product variants is high, which, in turn, may impact the system performance (quality and productivity). The paper considers the variety induced manufacturing complexity in manual, mixed-model assembly lines where operators have to make choices for various assembly activities. A complexity measure called "Operator Choice Complexity" (OCC) is proposed to quantify human performance of making the choices. The OCC takes an analytical form as an information-theoretic entropy measure of the average randomness in a choice process. Meanwhile, empirical evidences are provided to support the proposed complexity measure. Based on the OCC, models are developed to evaluate the complexity at each station, and for the entire assembly line. Consequently, complexity can be minimized by making systems design and operation decisions, such as error-proof strategies and assembly sequence planning.

1 INTRODUCTION

Traditional mass production was based on dedicated assembly lines where only one product model was produced in very large quantities. Such systems can achieve high productivity by using principles of economies of scales and work division between assembly stations [1]. However, in today's environment, where customers demand high product variety and short lead time, mass customization has been recognized as a new paradigm for manufacturing [2, 3]. Mass customization promises individualized products at mass production cost. As a result of such paradigm change, assembly systems must be designed to be responsive to customer needs while at the same time achieving mass production's quality and productivity. Mixed-model assembly lines (MMAL) have been recognized as a major enabler to handle increased variety. An MMAL typically takes the form of a flow line. The topics of effectively assigning tasks to stations and balancing the lines for the multiple product types have been active research areas [4].

Various industries are using mixed-model assembly lines. The variety of products offered in these lines has increased dramatically over the last decade. For example, in a typical automobile assembly plant, the number of different vehicles being assembled can reach tens of thousands in terms of the possible build-combinations of options.

Such an astronomical number of build-combinations undoubtedly present enormous difficulties in the design and operation of the assembly systems. It has been shown by both empirical and simulation results [5–7] that increased vehicle product variety has significant negative impact on the performance of the mixed-model assembly process, such as quality and productivity. Such impact can result from assembly system design as well as people performance under high variety. The effect from the latter persists since only limited automation can be implemented in the automobile final assembly [8–10]. Thus the questions presented here are two fold: how variety impacts people and system performance, and how to design assembly systems and organize production to allow high product variety without sacrificing quality and productivity.

One of the possible approaches to assess the impact of product variety on manufacturing system performance is to investigate how product variety *complicates* the mixed-model assembly process. However, only limited research has been done on defining manufacturing system complexity. For example, MacDuffie et al. [6] established *empirical* relationship between complexity and manufacturing system performance. They defined product mix complexity by looking at product variety (product mix and its structure) in assembly plants. According to the differences in the levels of product variety, three types of product mix complexity were defined in terms of empirical scores: Model Mix Complexity, Parts Complexity, and Option Complexity. By using statistical analysis, significant negative correlation between the complexity measures and the manufacturing performance was found. The result was based on the data from 70 assembly plants worldwide who participated in the International Motor Vehicle Program at M.I.T.

Besides empirical studies, attempts have also been made to *analytically* define complexity in manufacturing. For instance, complexity has once been associated with the amount of effort needed to make a part. The effort was quantified by a logarithmic function of the probability of achieving a certain geometric precision and surface quality in *machining* [11]. The function is widely known as Shannon's Information Entropy. Similarly, Fujimoto and Ahmed [12] defined a complexity index for *assembling*. The index takes the form of entropy to evaluate the assemblability of a product. The assemblability was defined as the uncertainty of gripping, positioning, and inserting parts in an assembly process. Also, complexity has been extended as a measure of uncertainty in achieving the specified functional requirements in axiomatic design [13].

Recently, complexity has been defined in an analytical form for manufacturing systems as a measure of how product variety complicates the process. Fujimoto et al. [14] introduced a complexity measure based on product structure using information entropy in different assembly process planning stages. By reducing the complexity, they claimed that impact of product variety on manufacturing systems could be reduced. Deshmukh et al. [15, 16] defined an entropic complexity measure for part mix in job shop scheduling. The complexity quantifies the difficulty associated with making scheduling decisions for the job shop, in which several types of products are manufactured simultaneously. An information-theoretic entropy measure of complexity is derived for a given combination and ratio of the part types. However, the complexity is not applicable for mixed-model assembly, which has a flow line or various hybrid configurations.

In summary, there is a general agreement that (i) product variety does increase the complexity in manufacturing systems, and (ii) information entropy is an effective measure of complexity. However, in order to analyze the impact of variety on manufacturing complexity in mixed-model assembly systems, one has to take into consideration the characteristics of the assembly system, such as system configuration, task to station assignment, and assembly sequences, etc. In addition, there is a lack of understanding on the mechanisms through which variety impacts manufacturing.

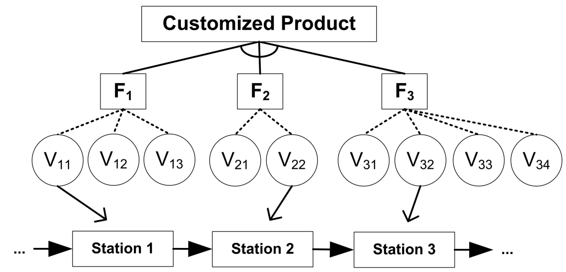

Figure 1. An illustration of a Product Family Architecture (PFA) and its mixed-model assembly line

To address the above issues, this paper defines new measure of complexity that depends on the integration of both product variety and assembly process information, and then develops models for evaluation of complexity in multi-stage mixed model assembly systems. The paper is organized as follows. Section 2 establishes a measure of operator choice complexity which results from the analysis of choices and choice process in mixed-model assembly operations. Moreover, the section also provides both theoretic and empirical justifications for the viability of the measure. Section 3 presents the model of system complexity for mixed-model assembly lines, where models at the station and system levels are investigated. Additionally, the influence of process flexibility and commonality is analyzed using numerical examples. Then potential applications for assembly system design by using the model are suggested in Section 4. Finally Section 5 concludes the paper and proposes future work.

2 MEASURE OF OPERATOR CHOICE COMPLEXITY

This section begins with a brief introduction to mixed-model assembly lines. Then it describes the choices and choice processes on the line to help theoretically define the choice complexity. Correspondences between theoretical definition and empirical results are discussed according to a group of studies from cognitive ergonomics.

2.1 Mixed-Model Assembly Line

Figure 1 illustrates an example of a product structure and its corresponding mixed-model assembly line. The product has three features (F_i); each feature has several variants (e.g., V_{ij} is the j^{th} variant of F_i). The product structure is represented by a Product Family Architecture (PFA) [17].

The PFA shows all the possible build-combinations of the customized products by combining the variants of each feature. For example, in Fig.1, we can have different end products by choosing one variant for every feature. Moreover, we represent the product mix information by a matrix **P**, where P_{ij} is defined as the demand (in percentage) of the j^{th} variant of the i^{th} feature. For instance, the **P** matrix for the product in Fig.1 is the following:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & 0 \\ p_{21} & p_{22} & 0 & 0 \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \tag{1}$$

Each row corresponds to the demands (in terms of mix ratio) for the variants of each feature, satisfying:

$$\sum_j p_{ij} = 1, \forall i \tag{2}$$

In mixed-model assembly, one variant of every feature is selected and assembled sequentially along the flow of the assembly line. For example, as depicted in Fig.1, V_{11} is chosen for F_1, V_{22} for F_2, and V_{32} for F_3. Quite often, this assembly process is accomplished manually. Operators at every stations must make correct choices among a number of alternatives. The choices include choosing the right part, tool, fixture, and assembly procedure for the variant.

2.2 Choices and Choice Processes

At each assembly station, the operator must choose the correct part from all possible variants according to customers' order. (The order is usually written on a production tag/manifest attached on the partially completed assemblage.) This process of selecting the right part is continuing during the day. To better understand the process, we define it *choice process*.

The choice process consists of a sequence of choices with respect to time. It can be modeled as a sequence of random variables, each of which represents choosing one of the possible alternatives. Mathematically, it can be considered as a discrete time discrete state stochastic process $\{X_t, t = 1, 2, \ldots\}$, on the state space $X_t \in \{1, 2, \ldots, M\}$, where t is the index of discrete time period, M is the total number of possible alternatives (parts) which could be chosen during each period. More specifically, $X_t = m, m \in \{1, 2, \ldots, M\}$, is the event of choosing the m^{th} alterative during period t.

In the simplest case, if the choice process is independent and identically distributed (*i.i.d.*), we can use a single random variable X (instead of X_t's) to describe the outcome of a choice. Furthermore, if we know all the alternatives of X and their distributions, the probability of a choice taking the m^{th} outcome is known as $p_m \triangleq \mathbf{P}(X = m)$, for $m = 1, 2, \ldots, M$. In the following discussions, we limit ourselves by assuming *i.i.d.* sequences.

2.3 Operator Choice Complexity

To characterize the operator performance in making choices, we define the term *operator choice complexity* (or *choice complexity*) as follows.

Definition: Choice complexity is the average uncertainty or randomness in a choice process, which can be described by a function H in the following form:

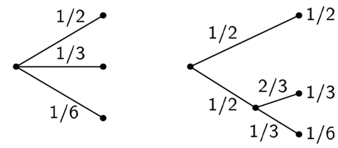

Figure 2. Decomposition of a choice from three possibilities [18]

$$H(X) = H(p_1, p_2, \ldots, p_M) = -C \cdot \sum_{m=1}^{M} p_m \cdot \log p_m \tag{3}$$

Theoretical Properties: The following seven properties of the function H as described in [18] make it suitable as a measure of choice complexity.

1) H is continuous in p_m, i.e., small changes in p_m should result in only small changes in choice complexity.

2) If p_m's are brought closer to each other, H would increase. Put alternatively, any change towards equalization of $p_1, p_2, \ldots, p_M$ should increase H. For a given M, H is a maximum and equal to $\log M$ when all p_i's are equal (i.e., $\frac{1}{M}$). In this case, H is a increasing function of M. This case is also intuitively the most uncertain situation to make a choice, since the operator is considered to be *non-informative* [19].

3) If a choice process is broken down into two successive stages, the original H is the weighted sum of the individual values of H. This is illustrated in Fig.2, where $H(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}) = H(\frac{1}{2}, \frac{1}{2}) + \frac{1}{2}H(\frac{2}{3}, \frac{1}{3})$.

4) $H = 0$ if and only if all the p_m's but one are zero, this one has the value of unity, i.e., $H(1, 0, \ldots, 0) = H(0, 1, \ldots, 0) = H(0, 0, \ldots, 1) = 0$. Thus only when we are certain of the outcome does H vanish and there exists no choice complexity. Otherwise H is positive.

5) H does not change when an additional alternative with no chance to happen is added into the original system.

6) H is a symmetrical function of $p_1, p_2, \ldots, p_M$, i.e., if the probabilities of choices are permuted among the alternatives, the amount of choice complexity does not change.

7) Suppose we have two successive choice processes, X (choices of tools) and Y (choices of fixtures) with M alternatives (numbered 1 to M) for the first and N for the second (numbered 1 to N). Let $p(x, y)$ be the probability of the joint event $\{X = x, Y = y\}$, where $x \in \{1, 2, \ldots, M\}$ and $y \in \{1, 2, \ldots, N\}$. The complexity of the joint choice is:

$$H(X,Y) = -\sum_{i=1}^{M} \sum_{j=1}^{N} p(x,y) \log p(x,y) \tag{4}$$

while,

$$H(X) = -\sum_{i=1}^{M} p(x,y) \log \sum_{j=1}^{N} p(x,y) \tag{5}$$

$$H(Y) = -\sum_{j=1}^{N} p(x,y) \log \sum_{i=1}^{M} p(x,y) \tag{6}$$

It is easy to show that,

$$H(X,Y) \leq H(X) + H(Y) \tag{7}$$

with equality achieved only if the two choices are independent, i.e., $p(x,y) = p(x)p(y)$, where $p(x) \triangleq P(X = x)$, and $p(y) \triangleq P(Y = y)$.

Thus the entropy function H possesses most of the desirable properties to be one of the possible measures of choice complexity.

2.4 Correspondences between Theoretical and Empirical Studies

Results similar to the seven theoretical properties can be found in human cognitive experiments. The experiments were conducted to assess human performance when making choices. Coincidentally, information entropy was found to be one of the effective measures. The performance of human choice-making activities was investigated by measuring average reaction times, i.e., how quickly a person can make a correct choice to a stimulus. One of the earliest studies was done by Merkei in 1885, described by Woodworth [20]. In the experiment, digits 1 through 5 were assigned to the fingers of the right hand and the Roman numbers I through V were assigned to the fingers of the left hand. On any given set of trials, the subject knew which of the set of stimuli would be possible (e.g., if there were three possible stimuli, they might be 3, 5, and V). Merkel studied the relationship between the number of possible stimuli and the choice reaction time (RT). His basic findings are presented in Fig.3(a), where the relationship between choice RT and the number of alternatives was not linear.

This relationship in Fig.3(a) has been further studied by a number of researchers since Merkel's original observations. Among them, the most widely known one was Hick [21]. He discovered that the choice RT is linearly proportional to the logarithm of the number of stimulus alternatives if all the alternatives are equal likely, see Fig.3(b), i.e.,

$$\text{Mean Choice RT} = a + b \cdot [\log_2 n] \tag{8}$$

where n is the number of stimulus-response alternatives, a and b are constants, which can be determined empirically by fitting a line to the measured data. This relation came to be known as *Hick's Law* (or *Hick-Hyman Law*), which was regarded as one major milestone in the area of cognitive ergonomics.

Coincidentally, the term $[\log_2 n]$ is exactly the information entropy calculated in Eqn.(3) if all the p_m's are equal, which follows from the result that the choice process is *i.i.d.* and all the

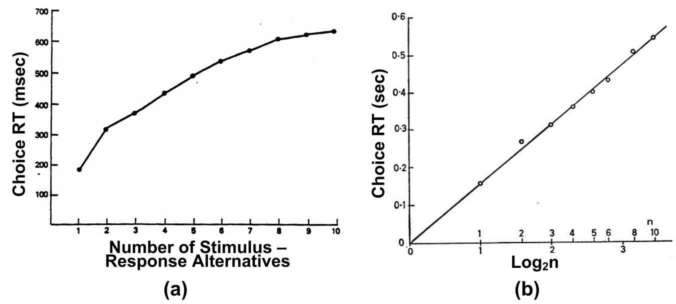

Figure 3. Mean choice RT as (a) a nonlinear function of the number of stimulus-response alternatives [20]; (b) a linear function of stimulus information, or $\log_2$ of the number of alternatives [21], reprinted from [22]

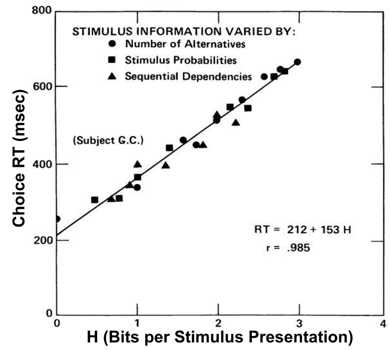

Figure 4. Choice RT for three different ways of manipulating the stimulus information H, reprinted from [24], using data from [23]

alternatives occur equal likely. The above analogy was first discovered by Hyman [23], where he concluded that, "The reaction time seems to behave, under certain conditions, in a manner analogous to the definition of information".

Hyman [23] also realized that, according to Shannon's definition of information entropy, he could change information content in the experiment by other means. Thus, in addition to varying the number of stimuli and letting each one of them occur equally likely in Hick's [21] experiment, he altered stimulus information content simply by (i) changing the probability of occurrence of particular choices, (ii) introducing sequential dependencies between successive choices of alternatives, see Fig.4. Thus, naturally enough, we can use H to replace the $[\log_2 n]$ term, Eqn.(**??**) becomes,

$$\text{Mean Choice RT} = a + b \cdot H \tag{9}$$

Because of the significance of this generalization, Hick's Law is also referred as the Hick-Hyman Law.

Moreover, it was suggested in Welford [22] that the information measure is adequate to assess human performance, since it provides a valuable means for combining reaction time and errors (i.e., speed and accuracy) into a single score.

2.5 Summary

According to both theoretical properties and empirical results, the entropy-based quantity H is suitable to measure *operator choice complexity* or *choice complexity*. Therefore, we propose to use the following form to quantify the value of choice complexity.

$$\text{Choice Complexity} = \alpha(a + b \cdot H), \alpha > 0 \tag{10}$$

The form is similar to that of the Hick-Hyman Law. It only differs in a positive scalar α, served as a weight to a specific choice process. In other words, the choice complexity is positive monotonic to the amount of uncertainty embedded in the choice process during the manual assembly process. Since Eq.(10) takes a simple linear form with constants α, a, and b, the only remaining part to be determined is the value of H when evaluating complexity. By incorporating information from product design, line design and operation, one can develop models and methodologies to quantify the information content in terms of the various operator choices in a mixed-model assembly process.

3 MODEL OF SYSTEM COMPLEXITY FOR MIXED-MODEL ASSEMBLY LINES

This section defines the operator choice complexity in the station level by simply extending the previous definition for a single assembly activity. Then complexity in the system level is examined after a unique propagation behavior of complexity is found. Moreover, process flexibility and commonality is taken into account when analyzing complexity. Finally a complexity model is proposed for multi-stage assembly systems.

3.1 Station Level Complexity

On a station, in addition to the part choice mentioned in Section 2, the operator may perform other assembly activities as well in a sequential manner, and some examples of these choices are briefly described as follows, see Fig.5.

Fixture choice: choose the right fixture according to the base part (i.e., the partially completed assemblage) to be mounted on as well as the added part to be assembled.

Tool choice: choose the right tool according to the added part to be assembled as well as the base part to be mounted on.

Assembly procedure choice: choose the right procedure, e.g., part orientation, approach angle, or temporary unload of certain parts due to geometric conflicts/subassembly stabilities.

According to Eqn.(10), we define the associated complexity at the station as part choice complexity, fixture choice complexity, tool choice complexity, and assembly procedure choice complexity respectively. All these choices contribute to the *operator choice complexity*.

Without loss of generality, we number the sequential assembly activities in Fig.5 from 1 to K and denote C_j as the total complexity at station j, which is a weighted sum of the various types of choice complexity at the station.

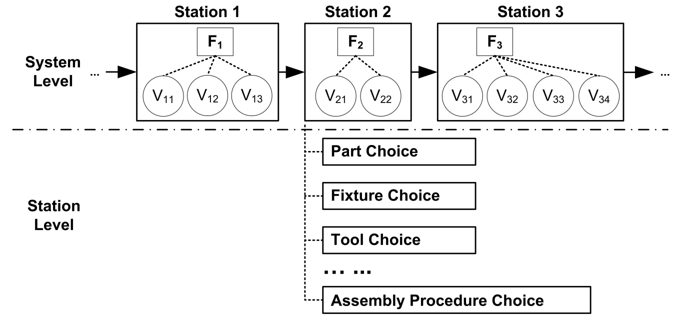

Figure 5. Choices in sequential assembly activities at one station

$$C_j = \sum_{k=1}^{K} \alpha_j^k (a_j^k + b_j^k \cdot H_j^k), \alpha_j^k > 0, k = 1, 2, \ldots, K \tag{11}$$

where α_j^k, are the weights determined by the task difficulty of the k^{th} assembly activity at station j; a_j^k's and b_j^k's are empirical constants depending on the nominal human performance similar to that of the choice reaction time experiments; H_j^k is the entropy computed from the variant mix ratio relevant to the k^{th} activity at station j. For simplicity and without loss of generality, we set $a_j^k = 0, b_j^k = 1, \forall j, k$. Then Eqn.(11) reduces to,

$$C_j = \sum_{k=1}^{K} \alpha_j^k H_j^k, \alpha_j^k > 0, k = 1, 2, \ldots, K \tag{12}$$

3.2 Propagation of Complexity

By Eqn.(12), complexity on individual stations is considered as a weighted sum of complexities associated with every assembly activities. Among them, some activities are caused only by the feature variants at the current station, such as picking up a part, or making choices on tools for the selected part. The complexity associated with such assembly activity is called *feed complexity*. However, the choice of fixtures, tools, or assembly procedures at the current station may depend on the feature variant that has been added at an upstream station. This particular component of complexity is termed as *transfer complexity*.

A formal definition of the two types of complexity is given below. Assume a current station j:

Feed complexity: Choice complexity caused by the feature variants added at station j.

Transfer complexity: Choice complexity caused by the feature variants already added at an upstream station, i.e., station i (i precedes j, denoted as $i \prec j$).

Transfer complexity exists because the feature variants added on the previous station i may affect the process of realizing the feature at station j, causing tool changeovers, fixture conversions, or assembly procedure changes.

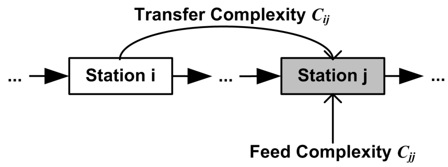

Figure 6. Complexity propagation scheme

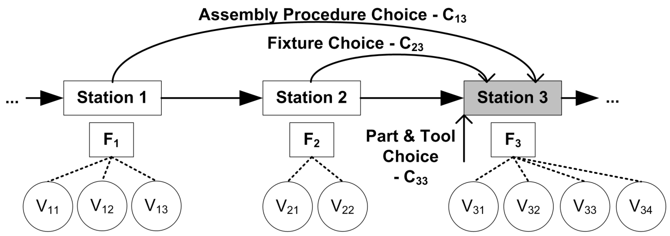

Figure 7. Complexity propagation of the example in Fig.1

The propagation behavior of the two types of complexity is depicted in Fig.6, where, for station j, the feed complexity is denoted as C_{jj} (with two identical subscripts), and the transfer complexity is denoted as C_{ij} (with two distinct subscripts to represent the complexity of station j as caused by an upstream station i). Thus the transfer complexity can flow from upstream to downstream, but not in the opposite direction. In contrast, the feed complexity can only be added at the current station with no flowing or transferring behavior.

Hence the total complexity at a station is simply the sum of the feed complexity at the station and the transfer complexity from all the upstream ones, i.e., for station j,

$$C_j = C_{jj} + \sum_{\forall i: i \prec j} C_{ij} \tag{13}$$

Compared with Eqn.(12), we may find equivalence relationships term by term between the two sets of equations. We illustrate this in the following section with examples.

3.3 Examples of Complexity Calculation

In this section, by continuing the example in Fig.1 which is redrawn in Fig.7, we demonstrate the procedures of calculating complexity at a station. More specifically, we will consider examples with or without process flexibility (and commonality) respectively.

3.3.1 Example without process flexibility In Fig.7, on the one hand, four sequential assembly activities are identified at station 3, complexity is expressed according to Eqn.(12) by assigning subscripts 1 to 4 as part choice complexity, fixture choice complexity, tool choice complexity, and assembly procedure choice complexity respectively.

$$C_3 = \alpha_3^1 H_3^1 + \alpha_3^2 H_3^2 + \alpha_3^3 H_3^3 + \alpha_3^4 H_3^4 \tag{14}$$

If we know from the process requirements at the station that,

1. One of the four parts, i.e., variants of F_3, is chosen according to customer order;
2. One of the four distinct tools is chosen according to the chosen variant of F_3;
3. One of the two distinct fixtures is chosen according to the variant of F_2 installed at station 2;
4. One of the three distinct assembly procedures is chosen according to the variant of F_1 installed at station 1.

On the other hand, the propagation scheme in the system level can be examined from the viewpoint of feed complexity (C_{33}) and transfer complexity (C_{13} and C_{23}), which is expressed according to Eqn.(13) as follows.

$$C_3 = C_{33} + C_{13} + C_{23} \tag{15}$$

There exist correspondences between Eqns.(14) and (15), or equivalently, Eqns.(12) and (13), which are shown below.

Part choice complexity: $\alpha_3^1 H_3^1 = \frac{\alpha_3^1}{\alpha_3^1+\alpha_3^3} C_{33}$
Tool choice complexity: $\alpha_3^3 H_3^3 = \frac{\alpha_3^3}{\alpha_3^1+\alpha_3^3} C_{33}$
Fixture choice complexity: $\alpha_3^2 H_3^2 = C_{23}$
Assembly procedure choice complexity: $\alpha_3^4 H_3^4 = C_{13}$

Note, $H_3^1 = H_3^3 = H_3$, where H_3 is the entropy associated with the variants added at station 3; and similarly, $H_3^2 = H_2$, $H_3^4 = H_1$. By complexity propagation, we have:

Feed complexity: $C_{33} = \alpha_3^1 H_3^1 + \alpha_3^3 H_3^3$
Transfer complexity: $C_{23} = \alpha_3^2 H_3^2$, $C_{13} = \alpha_3^4 H_3^4$

As a result, the sources of complexity are identified. The H terms are now easily calculated. That is, if an H term corresponds to the feed complexity, it is a function of the mix ratio of the current station; however, if an H corresponds to the transfer complexity, it is a function of the mix ratio of the station which is specified in the first subscript of its corresponding C_{ij}, i.e., station i.

Now, let us consider a numerical example, assume the **P** matrix in Eqn.(1) takes the following values.

$$\mathbf{P} = \begin{bmatrix} 0.5 & 0.2 & 0.3 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.3 & 0.3 & 0.2 & 0.2 \end{bmatrix} \tag{16}$$

Then,

$$\begin{aligned} H_3^1 &= H_3^3 = H_3 = H(0.3, 0.3, 0.2, 0.2) = 1.971 \text{ bits} \\ H_3^2 &= H_2 = H(0.5, 0.5) = 1 \text{ bit} \\ H_3^4 &= H_1 = H(0.5, 0.2, 0.3) = 1.485 \text{ bits} \end{aligned} \tag{17}$$

and,

$$C_3 = C_{33} + C_{13} + C_{23} = 1.971\alpha_3^1 + 1.971\alpha_3^3 + \alpha_3^2 + 1.485\alpha_3^4 \quad (18)$$

For simplicity, assuming $\alpha_3^1 = \alpha_3^2 = \alpha_3^3 = \alpha_3^4 = 1$, we finally obtain the total complexity at station j.

$$C_3 = 1.971 + 1.971 + 1 + 1.485 = 6.427 \text{ bits} \quad (19)$$

3.3.2 Influence of process flexibility and commonality

So far, we have illustrated in Eqns.(17) and (19) an example of calculating choice complexity with no flexibility or commonality in the manual assembly operations. However, flexibility is usually built into assembly systems such that common tools or fixtures can be used for different variants as to simplify the process. That is, flexible tools, common fixtures, or shared assembly procedures are adopted to treat a set of variants so that choices (of the tools, fixtures, and assembly procedures) are eliminated. Since fewer choices are needed, complexity reduces. However, not all the assembly processes can be simplified by flexibility strategies. Sometimes, flexible tools, common fixtures, or shared assembly procedure may require significant changes or compromise in product design and process planning, which is usually costly if not impossible. To characterize the impact of flexibility and commonality, i.e., to establish the relationship between product feature variants and process requirements, a *product-process association matrix* (denoted as Δ-matrix) is defined in the sequel.

We again use the example in Fig.7. At station 3, we consider fixture changeover, and it is denoted as the k^{th} assembly activity. Which fixture should be used in assembling F_3 at station 3 is determined by the variant of F_2 assembled previously at station 2. If no flexibility or commonality is present, fixture choice is needed at station 3 by observing feature F_2 (installed on station 2) according to the following rules,

Use fixture 1, if V_{21} is present;
Use fixture 2, if V_{22} is present.

Thus there are two states in the fixture choice process; the mapping relationship can be expressed in a Δ-matrix as follows.

$$\Delta_{23}^k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (20)$$

where Δ_{23}^k denotes the Δ-matrix for the k^{th} activity at station 3 associated with the variants at station 2; the columns are the states of the k^{th} activity at station 3, rows are the variants of the feature F_2 affecting the activity. The ones in the cells establish associations between the state in the column and the variant in the row.

A general form of the Δ-matrix for any assembly activity is given in as follows.

$$\Delta_{ij}^k = \begin{bmatrix} \delta_{1,1} & \delta_{1,2} & \ldots & \delta_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{n,1} & \delta_{n,2} & \ldots & \delta_{n,m} \end{bmatrix} \quad (21)$$

where,

$$\delta_{s,t} = \begin{cases} 1 & \text{Variant } s \text{ at station } i \text{ requires } k^{th} \text{ activity} \\ & \text{to be in state } t \text{ at station } j \\ 0 & \text{Otherwise} \end{cases} \quad (22)$$

m, n are the cardinality of states and variants respectively.

By definition, the Δ-matrix satisfies the following properties:

1. $\sum_{t=1}^{m} \delta_{s,t} = 1$, for $s = 1, 2, \ldots, n$;
2. $\sum_{s=1}^{n} \delta_{s,t} \geq 1$, for $t = 1, 2, \ldots, m$;
3. $n \geq m$.

Property (1) holds because one variant can lead to one and only one state. Property (2) holds because each state must be associated with at least one variant; otherwise, the column associated the empty state can be eliminated, and the size the matrix shrinks by 1. Lastly, property (3) holds because the maximal number of states cannot exceed the total number of variants. That is, in the extreme case of non-flexibility, each variant requires the characteristic to be in a distinct state, and the Δ-matrix becomes a unit matrix of dimension being the number of variants.

Consider the example in Fig.7 again, however, if a common fixture is adopted, the same fixture can be used no matter V_{21} or V_{22} is mounted on station 2. Thus, by definition, the Δ-matrix becomes simply:

$$\Delta_{23}^k = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \quad (23)$$

which should be reduced to:

$$\Delta_{23}^k = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (24)$$

By using the Δ-matrix, we are now capable of calculating the H terms when flexibility or commonality is present in the process. Define a vector $\mathbf{q}_{ij}^k = [q_1, q_2, \ldots, q_m]$, where $q_t, t \in \{1, 2, \ldots, m\}$ is the probability of the k^{th} activity being in state t at station j due to the variants added at station i, satisfying $\sum_{s=1}^{m} q_s = 1$. By the definition of the product mix matrix P in Eqn.(1) and the Δ-matrix in Eqn.(21), the following relationship is obtained:

$$\mathbf{q}_{ij}^k = [q_1, q_2, \ldots, q_m] = \mathbf{P}_{i\cdot} \times \Delta_{ij}^k \quad (25)$$

where $\mathbf{P}_{i\cdot}$ is the i^{th} row of the matrix $\mathbf{P}$, representing the mix ratio of the feature (i.e., feature F_2 in the example) assembled on station i. Thus, the corresponding H term is:

$$H_{jk} = H(\mathbf{q}_{ij}^k) = -\sum_{t=1}^{m} q_t \cdot \log_2 q_t \tag{26}$$

Revisit the example in Fig.7. When calculating the H term (H_{32}) corresponding to the fixture choice complexity, we have the following results for fixture choices at station 3.

Case 1 Use dedicated fixtures, i.e., a different fixture for each variant. By the Δ-matrix in Eqn.(20), we have:

$$\mathbf{q}_{23}^2 = \begin{bmatrix} q_1 & q_2 \end{bmatrix} = \mathbf{P}_{2\cdot} \times \Delta_{23}^2$$
$$= \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$
$$\Rightarrow H_3^2 = \textstyle\sum_{t=1}^2 q_t \cdot \log_2 \frac{1}{q_t} = 0.5 \log_2 \frac{1}{0.5} + 0.5 \log_2 \frac{1}{0.5} = 1 \text{ bit}$$

which duplicates exactly the result in Eqn.(17).

Case 2 Common fixture is used. By the Δ-matrix in Eqn.(24), we have:

$$\mathbf{q}_{23}^2 = \begin{bmatrix} q_1 & q_2 \end{bmatrix} = \mathbf{P}_{2\cdot} \times \Delta_{23}^2 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$$
$$\Rightarrow H_3^2 = \textstyle\sum_{t=1}^1 q_t \cdot \log_2 \frac{1}{q_t} = 1 \cdot \log_2 \frac{1}{1} = 0 \text{ bit}$$

Since fixture is common to the process of assembling F_3 with variants of F_2, no choice is needed.

Assume we have flexibility or commonality in fixture, tool, assembly procedures respectively, which is expressed by the Δ-matrices in Table 1. As a summary, the table also demonstrates a comprehensive numerical example to calculate complexity at station 3. The results show a reduced value of choice complexity compared with Eqn.(19) because of the flexibility or commonality added.

Table 1. Numerical Example of Complexity Calculation

No.	Activity	Δ-matrix	q-vector	H-term
1	Part Pick-up	$\mathbf{\Delta}_{33}^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	$\mathbf{q}_{33}^1 = \begin{bmatrix} .3 \\ .3 \\ .2 \\ .2 \end{bmatrix}^T$	H_3^1=1.971
2	Fixture Conversion	$\mathbf{\Delta}_{23}^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$	$\mathbf{q}_{23}^2 = \begin{bmatrix} 1 \end{bmatrix}^T$	H_3^2=0
3	Tool Changeover	$\mathbf{\Delta}_{33}^3 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$	$\mathbf{q}_{33}^3 = \begin{bmatrix} .6 \\ .4 \end{bmatrix}^T$	H_3^3=0.971
4	Assembly Procedure Change	$\mathbf{\Delta}_{13}^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$	$\mathbf{q}_{13}^4 = \begin{bmatrix} .8 \\ .2 \end{bmatrix}^T$	H_3^4=0.722
Total complexity at station 3 with equal weights				3.664

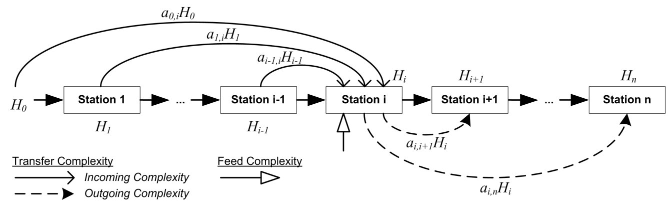

Figure 8. Propagation of complexity at the system level in a multi-stage assembly system

3.4 System Level Complexity Model

In general, consider an assembly line with n workstations, numbered 1 through n sequentially, see Fig.8. The mix ratio in Eqn.(1) is known. Using Eqn.(3), we can obtain the entropy H for the variants at each station according to their mix ratios.

The propagation of complexity in a multi-stage system can be analyzed by considering how the complexity of assembly operations (choices) at a station is influenced by the variety added at its upstream stations (*incoming complexity*), as well as how variants added at the station impact the downstream stations (*outgoing complexity*). The incoming complexity for stations in the system can be calculated in the following way:

Station 1: $C_1^{\text{in}} = C_{01} = a_{01}H_0$

Station 2: $C_2^{\text{in}} = C_{02} + C_{12} = a_{02}H_0 + a_{12}H_1$

......

Station i: $C_i^{\text{in}} = C_{0i} + C_{1i} + C_{2i} + \ldots + C_{i-1,i}$
$= a_{0i}H_0 + a_{1i}H_1 + a_{2i}H_2 + \ldots + a_{i-1,i}H_{i-1}$

......

Station n: $C_n^{\text{in}} = C_{0n} + C_{1n} + C_{2n} + \ldots + C_{n-1,n}$
$= a_{0n}H_0 + a_{1n}H_1 + a_{2n}H_2 + \ldots + a_{n-1,n}H_{n-1}$

where,

C_i^{in} – The incoming complexity of station i, $i = 1, 2, \ldots, n$;

H_i – Entropy of variants added at station i, $i = 1, 2, \ldots, n-1$;

H_0 – Entropy of variants of the base part;

a_{ij} – The coefficient of the impact (in term of choice complexity) on the assembly operations at station j due to the variants added at station i, i.e.,

$$a_{ij} \begin{cases} > 0 \text{ if } i < j \text{ and there exists impact from station } i \text{ to } j \\ = 0 \text{ if } i < j \text{ but no impact} \\ = 0 \text{ if } i \geq j \end{cases}$$

Or equivalently, by using a matrix representation, a comprehensive model can be obtained as follows:

$$\begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_n \end{bmatrix}^{\text{in}} = \begin{bmatrix} a_{01} & 0 & \ldots & 0 \\ a_{02} & a_{12} & \ldots & 0 \\ \vdots & & \ddots & 0 \\ a_{0n} & a_{1n} & \ldots & a_{n-1,n} \end{bmatrix} \times \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ H_{n-1} \end{bmatrix} \tag{27}$$

or,

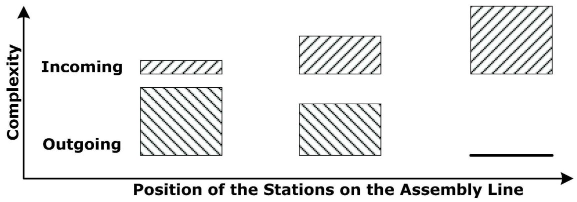

Figure 9. Incoming and Outgoing Complexity Charts

$$\mathbf{C}^{\text{in}} = \mathbf{A}^T \times \mathbf{H} \tag{28}$$

where $\mathbf{C}^{\text{in}}$ is the incoming complexity vector of size n for the system, with its i^{th} entry being the incoming complexity at station i, for $i = 1, 2, \ldots, n$; $\mathbf{A}$ is the characteristics matrix of size $n \times n$, which characterizes the interactions between stations (due to the feature variants added on the stations) in term of choice complexity; and $\mathbf{H}$ encapsulates product variety information including the number of variants and their distributions.

The outgoing complexity at station i, C_i^{out} is the amount of complexity flowing out of the station, influencing downstream operations. It can be analyzed by following a similar approach as used in Eqn.(27) or (28).

4 POTENTIAL APPLICATIONS

Once the propagation of complexity is understood and models developed, they can be applied to the design of mixed model assembly systems. Several potential applications are sketched below.

4.1 Performance Evaluation and Root Cause Identification Using Complexity Charts

Following the procedures in Section 3.4, we can analyze the incoming and outgoing complexity for each station and plot them against the station position in a multi-stage assembly system, see Fig.9. As a result, the stations with high incoming complexity are the potential stations where error-proofing strategies need to be provided to minimize the influence of complexity. Alternatively, flexible, automated assembly operations may be applied to minimize the impact of variety on operator performance.

In Fig.9, the outgoing complexity also shows how much influence the variants at one particular station have on its downstream operations. As a result, the outgoing complexity implies the root cause of the choice complexity in the system. Thus decisions from product design, such as commonality strategies, option bundling policies need to be considered to minimize outgoing complexity.

4.2 Influence Index and System Configuration Design

For any station j, once the values of incoming and outgoing complexity are found, we may define an index, called influence index, as follows:

$$I_j = \frac{C_j^{\text{out}}}{C_j^{\text{in}}} \tag{29}$$

The index quantifies how much relative influence the variants added at station j have in the operations of the other stations. To illustrate, in Fig.8, if every complexity streams have one unit of complexity, we can calculate the influence index for station j, $j = 1, 2, \ldots, n$ by simply counting the number of streams:

$$I_j = \frac{\text{\# of Outgoing Complexity Streams}}{\text{\# of Incoming Complexity Streams}} = \frac{n-j}{j} \tag{30}$$

Obviously,

$I_1 = n - 1$, the first station *potentially* has the maximal influence on the others;
$I_n = 0$, the last station has no influence on the others.

Thus, in such a sequential manufacturing process, the influence index is monotonically decreasing with respect to j. Hence we can conclude that operations at the later stations are more vulnerable to be affected by the variants assembled at the previous ones. Therefore, by wisely assigning assembly tasks (i.e., the functional features) onto stations, it is possible to prevent complexity streams from propagating. One of the intuitive approaches is to assign features with more variants to the stations of smaller influence index (downstream stations), and vise versa. In this aspect, the proposed complexity model implies the principle of "delayed differentiation", which already has become a common practice in industry [25].

However, by Eqn.(29), our model suggests that it is not sufficient by looking at the number of variants and the position where they are deployed according to the "delayed differentiation" principle. The evaluation of the impact of product variety on manufacturing complexity should also take into account the process flexibility and commonality built in the system. For instance, if all the variants from the upstream could be treated by the same flexible tools, shared fixture, and common assembly procedures in the downstream, variants can be introduced in the upstream without increasing system complexity. In this case, all the Δ-matrices for transfer complexity become column vectors with all ones in the column, indicating common process requirements for the feature variants in the product family. As a result, the transfer complexity vanishes.

Since different configurations have profound impact on the performance of the system [26], selecting an assembly system configuration other than a pure serial line may help reduce complexity. For instance, using parallel workstations at the later stages of a mixed-model assembly process reduces the number of choices on these stations if we can wisely route the variants at the joint of the ramified paths, see Fig.10. However, balancing

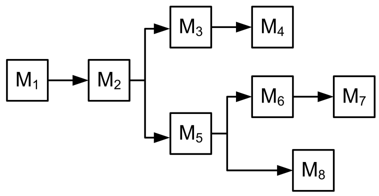

Figure 10. Possible configurations for mixed-model assembly systems. M_i's are machines in the system [27].

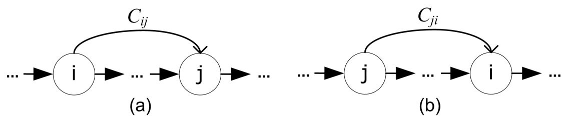

Figure 11. Differences in transfer complexity values for different assembly sequences: (a) Task i precedes j, which results in C_{ij}; (b) Task j precedes i, which results in C_{ji}, while C_{ij} and C_{ji} are not equal.

these types of manufacturing systems will be a challenge since the system configuration is not serial [27]. A novel method for task-machine assignment and system balancing needs to be developed to minimize complexity while maintaining manufacturing system efficiency.

4.3 Assembly Sequence Planning to Minimize Complexity

Assembly sequence planning is an important task in assembly system design. Since the assembly sequence determines the directions in which complexity flows, see Fig.11, proper assembly sequence planning can reduce complexity.

Generally, suppose we have a product with n assembly tasks, and the tasks are to be carried out sequentially in an order subject to precedence constraints. By applying the complexity model, we assume that the transfer complexity can be found between every two assembly tasks. Since only one of the two transfer complexity values in Fig.11 is effective (because only the upstream task/station has influence on the downstream ones) for one particular assembly sequence, an optimization problem can be formulated to minimize the system complexity by finding an optimal assembly sequence while satisfying the precedence constraints.

5 CONCLUSION AND FUTURE WORK

The paper proposes a measure of complexity based on the choices that the station operator has to make at the station level. The measure incorporates both product mix and process information. Moreover, models are developed for the propagation of complexity at the system level. The significance of this research includes: (i) mathematical models that reveal the mechanisms that contribute to complexity and its propagation in multi-stage mixed model assembly systems; (ii) understanding of the impact of manufacturing system complexity on performance; and (iii) guidelines for managing complexity in designing mixed-model assembly systems to optimize performance. Our future work will focus on the validation and applications of such model in assembly systems design and operations.

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