The Mathematics of Configuration

by Yoram Koren

Multi-stage manufacturing systems can be arranged in many ways, but those configurations classified as Reconfigurable Manufacturing Systems (RMS) have many advantages. Our rigorous mathematical study revealed a new method to calculate the possible RMS configurations based on the number of machines needed to satisfy throughput.

QUESTION – What is the optimal arrangement of machines to produce a product?

The configuration challenge is as follows: "Given: (a) a required part, (b) its daily volume (Q parts/day), (c) part processing time (t min/part), (d) machines, and (e) process precedent constraints, determine the system configurations that can process the part." The minimum number of machines (*N*) needed in a system is $N = (Q \times t)/T$, where T is the number of minutes per day. However, there is no general closed solution for the number of possible configurations with N machines, and this number increases exponentially, as seen in the table below.

Number of Machines Needed	Number of Possible Configurations	Number of RMS Configurations
2	2	2
4	15	8
6	170	32
8	2,325	128
10	**35,341**	**512**

Nevertheless, the number of practical configurations is much smaller, when we require that at each stage of the system (i) the process-plan must be identical, and (ii) machines that are doing the same set of tasks must be identical in their speed and functionality. Only two types of configurations satisfy these requirements: reconfigurable systems (RMS) and serial lines in parallel (cells).

The RMS Configurations are preferred because of their superior SCALABILITY and CONVERTIBILITY features. RMS configurations also provide more flexibility in balancing the system, which, in turn, may required a smaller number of machines to solve the configuration problem. Our thorough mathematical study yields that only for RMS-type configurations the number of possible configurations with *N* machines can be determined mathematically, by equations, and it is much smaller (see table).

RMS Configurations

Serial Lines in Parallel (Cells)

"The quality of research is very high." – 6th site visit report, 2002

The basic equations for calculating the number of possible RMS configurations are given as follows. The number K of possible configurations with N machines arranged in up to m stages is

$$K = \sum_{m=1}^{N} \binom{N-1}{m-1} = 2^{N-1}$$

The number K of possible configurations with N machines arranged in exactly m stages is

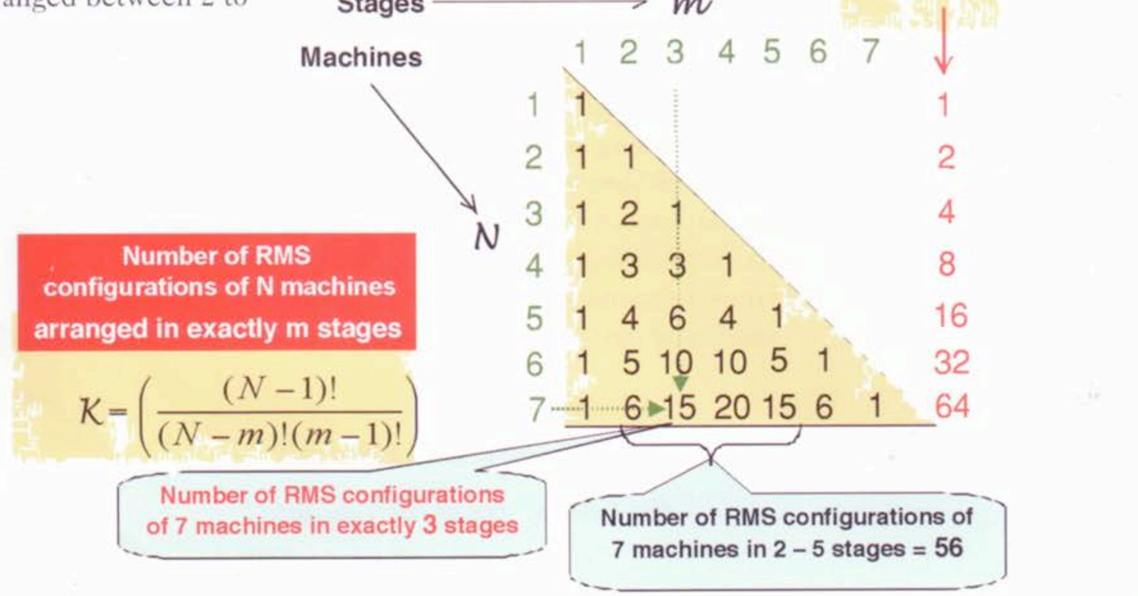
$$K = \left(\frac{(N-1)!}{(N-m)!(m-1)!} \right)$$

For example, for $N = 15$ and $m = 5$, yields $K = 1001$ configurations.

For the convenience of the system designer, the mathematical results of these two equations may be arranged in a triangular format, known as a Pascal triangle, shown below. The method of calculating the numerical value is defined by: The numerical value for N machines that are arranged in m stages is equal to the sum of: (the value for N-1 machines in m-1 stages) + (the value for N-1 machines in m stages).

The triangle allows the designer to immediately visualize the number of possible RMS configurations for N machines that are arranged in m stages. For example, there are 15 RMS configurations when 7 machines are allowed be arranged in exactly 3 stages.

The Pascal triangle also allows the designer to immediately calculate the number of possible RMS configurations for N machines that are arranged between i stages and j stages. The example shows that only 56 configurations exist with 7 machines arranged between 2 to 5 stages.



Pascal Triangles are helpful in calculating the number of RMS configurations for any number of machines.