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# A Simple Traction Control for Tracked Vehicles*

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**Abstract**

This paper introduces a simple traction control for a tracked vehicle (STCTV). This STCTV uses a zero-order sliding surface with time varying boundary layer thickness. This approach is proven to guarantee that the slip ratio converges to any pre-defined value asymptotically via Lyapunov Stability Theory. Simulation results confirm that STCTV allows the tracked vehicle to maintain maximum tractive force available in a frozen snow.

## 1. Introduction

On a low shear strength soil, such as frozen snow, a tracked vehicle may become unstable as the drive sprockets spin under excess engine torque during acceleration and lock during braking. This motivated us to investigate a possible traction control system to assure tracked vehicle operational stability on any kind of soil. The sliding controller introduced in this paper controls the engine torque and braking torque according to the measured vehicle speed and sprocket speed in such a manner that the slip ratio will converge to the pre-defined value to maintain the maximum tractive force available from the soil.

Fan, et. al., [1], Tan and Chin [2] investigated a traction control for a tracked vehicle and a car respectively. For their control law, they selected a first order switching indices to reduce chattering. However, such a control law generally requires the knowledge of the friction coefficient derivative and functional partial derivatives with respect to two state variables which are hard to obtain in real time. To overcome those difficulties, we introduce the STCTV. Our control laws are relatively simple to implement for they utilize a zero order sliding surface. To avoid chattering, we select a boundary layer with time-varying thickness. This sliding surface is guaranteed to be reached from any initial state in any region, as shown by both theoretical analysis and computer simulations.

In this paper, the sliding controller for tracked vehicle traction control is developed in Section 2. In Section 3, computer simulations are conducted to confirm the effectiveness of our sliding controller. Conclusions are given in Section 4. Symbols used in this paper are defined in Nomenclature.

Nomenclature

- $H$ total tractive force from ground
- $i$ slip ratio
- $i_d$ desired slip ratio
- $J_w$ moment of inertia of rotating parts referred at the wheel
- $M$ tracked vehicle mass
- $r$ sprocket radius
- $R_c$ compaction resistance
- $R_d$ aerodynamic resistance
- $R_r$ internal resistance
- $T$ net torque applied to sprocket
- $T_w$ sprocket friction torque
- $V_a$ actual vehicle velocity
- $V_t = r\omega$ theoretical vehicle velocity
- $\omega$ sprocket angular velocity

## 2. Sliding Controller Design

In this section, a controller is proposed such that the tracked vehicle will operate in the vicinity of $i = i_d$ so that maximum tractive force will be available in the specific soil. For purposes of example, the road surface will be assumed to be frozen snow with $i_d = 0.2$.

In order to design a sliding controller, we need to pick-up a well-behaved switching surface S=0. Then we should select the feedback control law such that $S^2$ remains a Lyapunov-like function of the closed loop system, despite the presence of model imprecision and of disturbances. A feedback control law is selected to guarantee that the squared 'distance' to the surface, as measured by $S^2$, decreases along all system trajectories. Thus, it constrains trajectories to point towards the sliding surface S=0. In particular, once on the sliding surface, the system trajectories remain there. The appropriate selection of sliding surface and feedback control law are the keys to the sliding controller design.

For simplicity, we select a zero order sliding surface shown in equation (1).

$$S = (i - i_d) \bullet \max(V_a, V_t) \tag{1}$$

where

$$\max(V_a, V_t) = \begin{cases} V_a & \text{if } V_a \geq V_t \\ V_t & \text{if } V_a < V_t \end{cases}$$

For convenience, the simplified longitudinal vehicle model introduced in [1] may be rewritten as:

$$\dot{x}_1 = -g_1(x_1) + b_1 H \tag{2}$$

$$\dot{x}_2 = -g_2(x_2) - b_2 H + b_3 T \tag{3}$$

where

$$x_1 = V_a \qquad x_2 = V_t$$

$$g_1(x_1) = \frac{R_c + R_d + R_r}{M}$$

$$g_2(x_2) = \frac{T_w r}{J_w} \qquad b_1 = 1/M$$

$$b_2 = \frac{r^2}{2J_w} \qquad b_3 = \frac{r}{J_w}$$

We seek a feedback control law that will guarantee that the system trajectory moves towards the sliding surface and stays on it. As proven in the Appendix, for the vehicle during acceleration, the sliding surface S=0 can be reached from any initial state in any region if

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$$T = \frac{\dot{x}_1/(1-i_d) + g_2 + b_2 H}{b_3} - k\,\mathrm{sgn}(S) \qquad (4)$$

where

$$k = \frac{\eta}{(1-i_d)b_3} \qquad \mathrm{sgn}(S) = \begin{cases} 1 & \text{if } S > 0 \\ -1 & \text{if } S < 0 \end{cases} \qquad \eta > 0$$

Similarly, for the vehicle during braking, the sliding surface S=0 can be reached from any initial state in any region if

$$T = \frac{(1+i_d)\dot{x}_1 + g_2 + b_2 H}{b_3} - k\,\mathrm{sgn}(S) \qquad (5)$$

where

$$k = \frac{\eta}{b_3} \qquad \eta > 0$$

Equation (4) is the feedback control law we should implement for anti-spin acceleration. Equation (5) should be implemented for anti-lock braking. When the sliding controller regulates the net torque T such that equation (4) or (5) is satisfied, S=0 is guaranteed and the slip ratio is guaranteed to converge to the desired slip ratio, despite the presence of model imprecision or disturbances.

However, the switch control is too oscillatory with zero order sliding mode. To reduce the chattering, we can smooth out the control discontinuity in a thin boundary layer neighboring the switching surface. The smoothing of control discontinuity essentially assigns a lowpass filter structure to the local dynamics of the variable S. In our smoothed implementation, the term ksgn(S) in equations (4) and (5) is actually replaced by $\bar{k}\,\mathrm{sat}(S/\Phi)$, where

$$\bar{k} = k - \dot{\Phi} \qquad \dot{\Phi} + \lambda\Phi = k$$

$$\mathrm{sat}(x) = \begin{cases} x, & |x| < 1 \\ \mathrm{sgn}(x), & |x| \ge 1 \end{cases}$$

### 3. Computer Simulations

In order to verify the control law suggested by equation (4), a computer simulation for anti-spin acceleration was conducted. We wrote a simulation software package specifically for the traction control of a tracked vehicle to handle the complex nonlinearity. The same vehicle parameters are used as in [1].

For all the simulation results presented here, $\lambda$ is set to 8 Hz and $\eta$ is set to 16000. We selected a large $\eta$ value to increase the convergence of the slip ratio to its desired value. The desired slip ratio is 20%. The maximum net torque is set at 10000 N-m and the maximum net torque rate is set at 100000 N-m/s. Initial speeds for both tracked vehicle and driving sprockets are 0 m/s. Initial engine torque is set at 5800 N-m which is larger than its steady state value. Figure 3 shows the simulation results for slip ratio versus time. As we apply a very large torque initially, the system trajectory jumps to the unstable region with a slip ratio of 0.7. The tracks would continue to spin without any traction control. With traction control, the spinning is stopped, the system trajectory returns to the stable region, and the slip ratio approaches the desired value immediately. Both the theoretical velocity and the actual velocity increase with the maximum acceleration permissible under a given terrain.

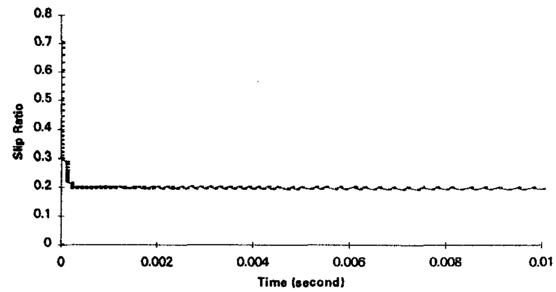


Figure 1. Slip Ratio versus Time

In order to implement our control laws on real systems, we need to measure the sprocket speed and vehicle speed. Traction H can be estimated from the vehicle acceleration or calculated with equation (7) in [1]. In spite of estimation errors, equations (4) and (5) should be valid due to the robust properties of the sliding controller as long as the estimation errors are bounded.

### 4. Conclusion

The STCTV is developed with a zero order sliding surface. Such a controller is relatively easy to implement for it does not require the measurement of the tractive force derivative and partial derivative with respect to state variables. The STCTV is proven to guarantee that the slip ratio will reach the desired value despite the model imprecision and disturbance. Simulation results verified this conclusion.

### References


[1] Fan, Zhejun, Wehe, D., and Koren, Y., "Traction Control and Modeling of Tracked Vehicles," SAE Technical Paper 942375.

[2] Tan, Han-Shue and Chin, Y.K., "Vehicle Traction Control: Variable Structure Control Approach", Journal of Dynamic Systems, Measurements, and Control, Vol. 113, No. 2, June 1991, pp. 223-230.


### Appendix

To prove the assertion associated with equation (4), we can apply Lyapunov Stability Theory. Take derivative of equation (1) to obtain:

$$\dot{S} = (1-i_d)\dot{x}_2 - \dot{x}_1 \qquad (6)$$

Substituting equation (3) into equation (6) yields:

$$\dot{S} = (1-i_d)[-g_2 - b_2 H + b_3 T] - \dot{x}_1 \qquad (7)$$

Substituting equation (4) into equation (7), we get:

$$\dot{S} = -\eta\,\mathrm{SGN}(S) \qquad (8)$$

If we select $S^2$ to be a Lyapunov function $V = S^2$ (9)

then $\dot{V} = S\dot{S}/2$ (10)

Substituting equation (8 ) into the above expression, one obtain

$$\dot{V} = -\eta S \bullet \mathrm{SGN}(S)/2 < 0 \text{ for } \eta > 0 \qquad (11)$$

Therefore, our proposed control law guarantees global stability. $S^2$ decreases along all system trajectories until S=0.

Similarly, we can prove the assertion associated with equation (5) when we substitute equation (5) into $\dot{S}$ for anti-lock braking case.