Design of a Digital Loop for Numerical Control

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Abstract—This paper describes a method for designing a digital loop for numerical control (NC) comprising an up–down counter, a digital-to-analog converter (DAC), a servo motor, and a digital encoder. The loop is analyzed and the open-loop gain is determined. The number of counter stages required for a given speed interval is also calculated. An example is presented to demonstrate the design procedure.

I. Introduction

NUMERICAL CONTROL (NC) systems for machine tools are of two types: point-to-point and contouring. In a point-to-point system (e.g., a drilling machine), the path of the cutting tool and its velocity while traveling from one point to the next have no particular significance. In a contouring system, the cutting tool is working while the axes of motion are moving, for example, as in a milling machine. In a multiaxes contouring control system, accurate speed control of each axis of motion is of extreme importance, as any speed deviation causes a path deviation and affects the shape of the workpiece.

In a contouring system, each axis of motion is controlled by a closed loop. The loop to be considered here is of a digital type with a sequence of pulses as the reference signal. The axis velocity is proportional to the pulse frequency and its position to the number of reference pulses. The feedback device in the digital control loop can be either a resolver [1], [2] or an optical encoder [3], [4]. An encoder-based control system consisting of an up–down counter, a digital-to-analog converter

Manuscript received June 10, 1977; revised April 7, 1978.

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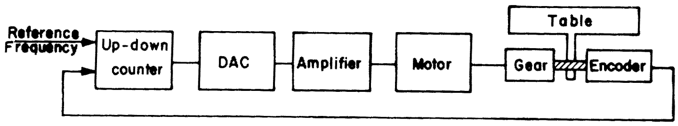

Fig. 1. Digital loop for numerical control of machine tools.

(DAC), a power amplifier, and a servo dc motor is shown in Fig. 1.

One design method proposed for a similar type of digital loop is based on minimizing the number of counter stages by determining an optimal gain for each stage while keeping a constant form factor in the armature [5]. Such a design, however, results in a loop gain which is varied discretely and consequently depends upon the actual speed of the motor. This approach cannot be implemented in contouring systems, which must operate linearly to maintain path accuracy [6].

In the present study, a method for selecting the loop parameters to meet the accuracy requirements is proposed. It is shown that the most important factors in designing a digital loop for NC are the choice of the correct number of stages in the counter and the appropriate gain of the amplifier.

II. Loop Operation

The principle of this digital loop is a comparison between two sequences of pulses. The up-down counter compares the frequency and the phase of the input pulses with the frequency provided by the encoder and generates a number representing the instantaneous position error in pulse units. This number is converted by the DAC to a voltage which is amplified and applied to the motor. The motor rotates in the direction that reduces the error. If a constant input frequency is applied, the encoder frequency in the steady-state is identical to the input frequency except for a finite pulse and phase difference, which are necessary to generate the corrective error voltage to rotate the motor.

A typical output signal of the counter at steady-state for a constant speed is shown in Fig. 2. Generally, the position of the counter at steady-state is not constant and its reading varies between two successive values (e.g., between three and four pulses). At high input frequencies, the motor smoothes the error signal and follows the average value, but with low input frequencies the motor moves in steps. When the input frequency varies with time, the average number in the counter also becomes a time dependent. In this case, the error signal is not constant but depends upon how the input frequency varies.

The digital control loop illustrated in Fig. 1 can rotate the motor in only one direction. In practice, a digital loop can rotate the motor in both directions, for which additional circuits are required [4].

An input circuit preceding the counter directs reference and feedback pulse sequences. For one direction of rotation, the count is increased by the reference and reduced by the feedback, and vice versa for reversed rotation. The input circuit also eliminates a simultaneous appearance of pulses in both channels, which would interfere with the counting. Information about the actual direction of rotation is obtained from the encoder which feeds two sequences of square waves in

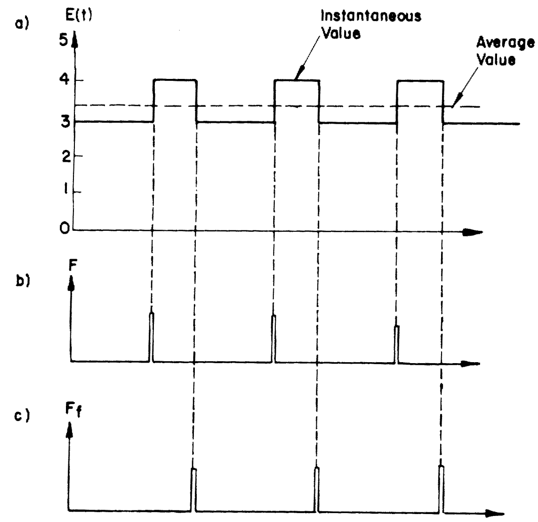

Fig. 2. The counter output for constant speed rotation.

quadrature into a direction sensing circuit. This provides a signal indicating the actual direction of rotation and ensures that the loop is of a negative feedback type. By applying an appropriate bias voltage, the DAC output can be varied over a negative-to-positive voltage range to control the speed and direction of rotation of the motor.

III. Mathematical Analysis

The digital control loop can be analyzed by using Laplace transform techniques. Each of the gain terms associated with the blocks of Fig. 1 can be defined as follows:

K_c conversion gain of DAC (V/pulse),
K_a amplifier voltage gain,
K_m motor constant (rev/s/V),
K_g gear ratio,
K_e encoder gain (pulses/rev).

The counter converts frequencies to a pulse number and phase difference, both being the integral of frequency. Therefore, the counter functions as an integrator in the loop, and its output $E(s)$ is given by

$$E(s) = \frac{F(s) - K_g K_e W(s)}{s} \qquad (1)$$

where F is the input frequency to the loop.

The Laplace transformed speed of the motor for negligible armature inductance is written as [7]

$$W(s) = \frac{K_m V(s) - K_t T(s)}{1 + s\tau} \qquad (2)$$

where τ is the mechanical time constant of the motor, which is proportional to the total amount of inertia of the driven system

$$\tau = K_t J. \qquad (3)$$

The complete model of the loop is presented in Fig. 3. The proportionally constant in (3) can be written

$$K_t = RK_m/K_1 \qquad (4)$$

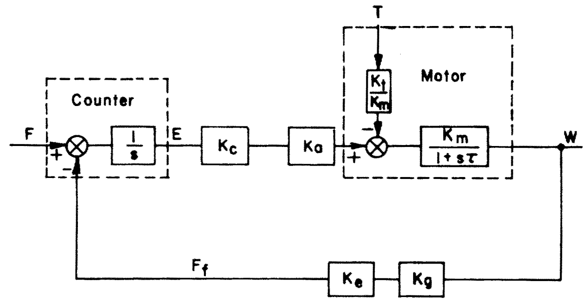

Fig. 3. Block diagram of the digital control loop.

where R is the armature resistance and K_1 is the torque constant. Most NC systems contain an additional internal loop consisting of the motor and a tachogenerator as a second feedback device. This internal loop has a mathematical representation similar to the one in (2) [8].

The forces, and consequently the torques, in most machining operations are almost linearly on the feed rate. In addition, there is a small torque in the motor due to friction losses in the driving system. Therefore, for contouring applications, the total torque is assumed to consist of a main torque for cutting which is proportional to the motor speed and an additional constant torque T_c representing the losses

$$T(s) = K_l W(s) + T_c(s). \tag{5}$$

The parameter K_l depends on the material being machined and the cutting geometry. Substituting (5) into (2), the Laplace transformed velocity can be written

$$W(s) = \frac{\beta K_m V(s) - \beta K_t T_c(s)}{1 + s\tau'} \tag{6}$$

where

$$\tau' = \beta\tau \tag{7}$$

and β is the fraction

$$\beta = 1/(1 + K_t K_l). \tag{8}$$

Combining (1) and (6) gives the closed-loop response

$$W(s) = \frac{\beta K_c K_a K_m F(s) - \beta K_t s T_c(s)}{\tau' s^2 + s + \beta K} \tag{9}$$

where K is the open-loop gain

$$K = K_c K_a K_m K_g K_e. \tag{10}$$

From (9), it is seen that the digital loop behaves as a second-order servo system with the characteristic equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0 \tag{11}$$

where the damping factor is

$$\zeta = 1/(2\sqrt{\beta K \tau'}) = 1/(2\beta\sqrt{K\tau}) \tag{12}$$

and the natural frequency is

$$\omega_n = \sqrt{K/\tau}. \tag{13}$$

Eliminating τ from (12) and (13) gives

$$\omega_n = 2\beta\zeta K. \tag{14}$$

By substituting (9) into (1), the Laplace transform of the counter position becomes

$$E(s) = \frac{(1 + s\tau')F(s) + \beta K_t T_c(s)}{\tau' s^2 + s + \beta K}. \tag{15}$$

For constant input frequency at the steady-state, the position of the counter is

$$E_{ss} = \frac{F}{\beta K} + \frac{K_t T_c}{K}. \tag{16}$$

Clearly, E_{ss} in this equation cannot be an integer throughout the whole speed interval. At those few speeds where it is an integer, the motor input voltage is direct. Usually, the counter output consists of a direct component and rectangular pulses as was shown in Fig. 2.

The corresponding time responses of W and E, for a constant input frequency, are found by an inverse Laplace transform of (9) and (15)

$$\begin{aligned} W(t) = {} & [1 - Q(t)\sin(\omega_d t + \phi)]F/K_g K_e \\ & - 2\zeta\beta K_t T_c Q(t)\sin\omega_d t \end{aligned} \tag{17}$$

$$\begin{aligned} E(t) = {} & [1 - Q(t)\sin(\omega_d t + \phi)]F/\beta K \\ & + (F/\omega_n)Q(t)\sin\omega_d t \\ & + [1 - Q(t)\sin(\omega_d t + \phi)]K_t T_c/K \end{aligned} \tag{18}$$

where

$$\omega_d = \sqrt{1 - \zeta^2 \omega_n}$$

$$\phi = \arccos\zeta$$

$$Q(t) = \exp(-\zeta\omega_n t)/\sqrt{1 - \zeta^2}.$$

For a negligible T_c, equation (18) has a maximum at

$$\omega_d t = \arctan(-\sqrt{1 - \zeta^2}/\zeta) = \pi - \phi$$

which recalls an overshoot of

$$P = \frac{F}{2\zeta\beta K}\exp\left[-\frac{\zeta(\pi - \phi)}{\sqrt{1 - \zeta^2}}\right]. \tag{19}$$

The overshoot percentage of the counter versus the damping factor is plotted in Fig. 4. This graph can be used to calculate the maximum capacity of the up-down counter.

IV. Design for Constant Input Frequency

Design of the digital control loop for an NC machine tool is generally performed for given servo motor characteristics, kinematic limitations of the machine tool, and precision requirements for the machining process.

Selection of servo motors for NC machine tools is based upon power and torque requirements for machines which fix the maximum motor speed W_m and motor constant K_m. Likewise, the maximum allowable feed rate for each axis of motion is dictated by kinematic considerations of the machine tool. Since each reference pulse is equivalent to the position resolution unit of the machine, this feed-rate limitation is equivalent to limiting the reference pulse frequency. The position resolution unit henceforth is called the basic length unit

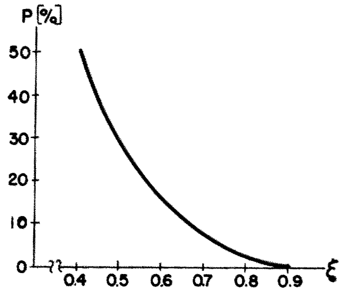

Fig. 4. The overshoot percentage of the counter versus damping factor.

(BLU) and is equivalent to one pulse weight in length units. The maximum reference frequency in pps is given by

$$F_m = \mathrm{FRM}/60/\mathrm{BLU} \tag{20}$$

where FRM is the maximum feed rate in BLU per minute.

For example, a typical system with a maximum allowable feed rate of 30 in/min and a BLU = 0.0001 in/pulse has a maximum reference frequency of 5000 pps according to (20).

At the maximum reference frequency, the axis moves at its highest designed feed rate which corresponds to a certain motor speed W_0. The maximum motor speed W_m will have to be greater than W_0 to ensure linear operation, to accomodate overshoots, and to maintain a safety margin. Therefore, the maximum feed rate will be achieved with a motor speed of W_0 rev/s related to W_m as

$$W_0 = \alpha W_m \tag{21}$$

where α is a constant which is typically about 0.8. The relationship between this speed and the maximum frequency is

$$F_m = K_g K_e W_0. \tag{22}$$

Another parameter which is given is the lead-screw pitch (LP) given in mm/rev (SI units) or in threads per inch (TPI, in English units), i.e., LP = 1/TPI. This parameter, together with the resolution unit, gives the encoder gain

$$K_e = \frac{\mathrm{LP}}{\mathrm{BLU}} \left[\frac{\text{pulse}}{\text{rev}}\right]. \tag{23}$$

Typical values in SI units are LP = 5 mm/rev and BLU = 0.01 mm/pulse, resulting K_e = 500 pulses/rev.

Since K_e, F_m, and W_0 are known, the gear ratio K_g is determined from (22).

The open-loop gain K is calculated from (12) for a given time constant τ of the motor when coupled to the machine table. The fraction β in (12) is obtained from (8). For the purpose of gain design, the largest β ($\beta = 1$) should be considered. Designing for a damping factor of 0.707, the open-loop gain is

$$K = \frac{1}{2\tau}. \tag{24}$$

The next step is to determine the number of stages of the up-down counter and the DAC, which depends upon the maximum values of the variables. The counter must be capable of handling a motor speed of W_m. The corresponding input frequency, as derived from (21) and (22), is F_m/α which, together with (16), gives

$$E_{\max} = \frac{F_m}{\alpha\beta K} + \frac{K_t T_c}{K}. \tag{25}$$

In this case, the smallest value of β for full loading conditions is used.

The number of stages n of the up-down counter, and the DAC (including one stage serving as a sign bit, indicating the direction of rotation) is derived from

$$E_{\max} \leqslant 2^{n-1} - 1. \tag{26}$$

In contouring systems, each control loop must operate linearly to maintain path accuracy. Condition (26) guarantees that the counter will not become either full or empty, thus avoiding nonlinear operation. For negligible friction torque T_c, a simplified relationship is obtained

$$2^n > 2F_m/\alpha\beta K. \tag{27}$$

The DAC gain K_c depends on its maximum output voltage U_c and the number of stages

$$K_c = U_c/2^{n-1}. \tag{28}$$

The final step in the design is to determine the amplifier gain from (10) (as all other gains have been calculated already). For full-range operation, the maximum effective input voltage to the amplifier U_a is related to U_c as

$$U_a/U_c = 2E_{\max}/2^n. \tag{29}$$

V. Design Example

This design procedure is illustrated for the digital loop of an NC system which was applied for a large lathe.

The feed drives selected for the lathe were dc servo motors rated at 120 lb in (13.6 N · m) nominal torque. Technical specifications of the motor are given in Table I. The lathe is equipped with 10-mm/rev lead screws and a resolution of BLU = 0.01 mm is required. The maximum required feed rate is 1200 mm/min (~27 in/min) which corresponds to 2000 pps according to (20).

From (23), it is concluded that an encoder of 1000 pulses/rev is required. This frequency was achieved with an encoder of only 250 cycles/rev by having two channels in quadrature. Two channels are required for the direction sensing circuit, and by using the falling and rising edges of both waves as pulse sources, the encoder fundamental frequency is multiplied by a factor of 4.

For the nominal recommended motor speed of 720 rev/min, the required gear ratio from (22) is

$$K_g = (2000 \times 60)/(720 \times 1000) = \tfrac{1}{6}. \tag{30}$$

For designing the optimal gain according to (24), the open-loop time constant τ has to be found. Our specific system contains an additional internal velocity loop, consisting of the amplifier (a PWM type), the motor, and a tachogenerator as a second feedback device. A typical experimental response of

TABLE I
TECHNICAL SPECIFICATION OF SERVO MOTOR [9]

Nominal Torque	T	120 in-lb
Nominal Speed	W_0	720 rpm
Torque Constant	K_1	10.27 in-lb/amp
Voltage Constant	K_m	0.862 rad/sec/volt
Mech.Time Constant	τ_m	11.97 msec
Armature Resistance	R	0.75 ohms
Maximum Speed	W_m	1000 rpm
Peak Stall Torque	T_p	1026 in-lb
Friction Torque	T_f	5.0 in-lb
Moment of Inertia	J_m	0.19 in-lb-sec^2

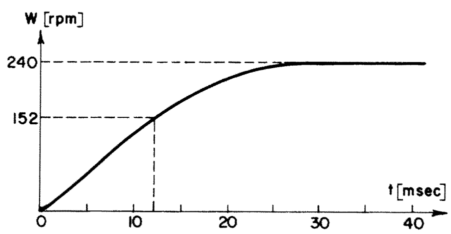

Fig. 5. Experimental open-loop response.

this loop is shown in Fig. 5. For simplicity, this time response is treated like that of a first-order system. The characteristic time constant at 63 percent (152 rev/min) of the steady-state is $\tau = 12$ ms which, from (24), gives $K = 42\ \mathrm{s}^{-1}$.

The speed dependent torque coefficient in (5) can be estimated by applying a full-load (120-in · lb) condition at the nominal speed

$$K_l = 120 \times 60/(720 \times 2\pi) = 1.59\,[\mathrm{in \cdot lb/rad/s}].$$

At the normal operating speeds, the magnitude of the constant torque is negligible compared with the rated torque.

Applying (4) with the data given in Table I gives $K_t = 0.063$ rad/s · in · lb and, consequently, the smallest β is determined from (8) yielding $\beta = 0.91$. The factor α, the ratio between the nominal and maximum speeds, is $\alpha = 0.72$. As a consequence, the maximum position of the counter, at steady-state, as given by (25) is $E_{\max} = 74$, which in turn dictates an 8-bit counter and an 8-bit DAC according to (26). Actually, such a DAC has a larger capacity than required and, therefore, can accommodate unexpected overloads. A DAC with a maximum output voltage of ± 10 V was chosen yielding, as a consequence, a gain of $K_c = 10/127$. Note that the maximum effective voltage of the amplifier as given from (29) is 5.8 V.

The design system has been constructed and tested on an NC lathe. The experimental results, presented in Fig. 6, clearly demonstrate that a practical optimal gain almost coincides with the calculated one of $K = 42\ \mathrm{s}^{-1}$. Lower gains yield sluggish response while high gains cause oscillations.

Finally, the output of the DAC, converted to counter pulses, was recorded at low speeds. A typical plot taken at 30 mm/min (1.2 in/min) without cutting load is presented in Fig. 7. This

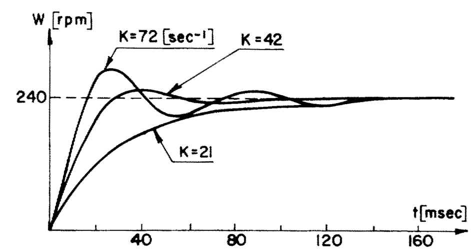

Fig. 6. Closed-loop response with the open-loop gain as parameter.

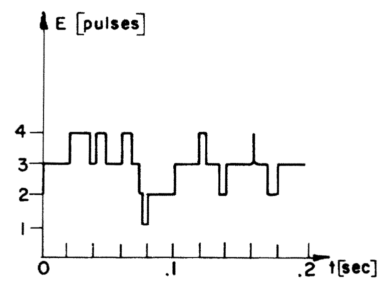

Fig. 7. Typical error at low velocity ($K = 21\ \mathrm{s}^{-1}$).

plot is quite similar to the theoretical one given in Fig. 2, however, due to unconstant friction torques, more variations are appearing around the average value of the error.

VI. CONCLUSIONS

A method is presented for designing a digital loop for NC control. The digital loop has an up-down counter as the equalizer and an encoder as the feedback device. The design procedure permits the determination of the counter stages and the various loop gains. The application of this method is illustrated using an NC lathe.

NOMENCLATURE

- E Average position of the counter.
- F Input frequency.
- F_m Maximum input frequency.
- K Open-loop gain.
- n Number of counter stages.
- s Laplace variable.
- T Torque.
- T_c Constant friction torque.
- W Speed of motor.
- W_m Maximum speed of motor.
- α Ratio between nominal and maximum speeds.
- τ Mechanical time constant.
- ζ Damping factor.
- ω_n Natural frequency.

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Analysis and Synthesis of Waveform Generators in the Phase Plane

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Abstract—A new class of waveform generators can be analyzed and synthesized using the phase plane. The technique is based upon generating closed paths in the phase plane such that the variable values as a function of time are the waveforms desired. Very accurate square, triangle, sawtooth, and pulse waveforms are generated using these concepts. Circuit implementations are proposed. Standard IC's containing four operational amplifiers allow inexpensive construction of these versatile oscillators, such that the component cost is competitive with common generators. Experimental results were very good.

I. Introduction

THE AVAILABILITY of low-cost high-performance operational amplifiers offers the possibility of efficiently building waveform generators such as sine, square, triangle, sawtooth, or many other types desired [1], [2]. By using phase-plane concepts, a technique becomes available for designing waveform generators. The quadrature oscillator [1] for generating sinewaves uses this concept, although very little attention has been devoted to the phase-plane approach. Thus the idea, while not entirely new, apparently has received little attention. It is the purpose of this paper to generalize this concept and use it to generate square, triangle, sawtooth, and pulse waveforms.

The phase plane is a powerful technique that is well known for nonlinear differential equation and nonlinear control system solutions [3], [4]. It allows one to observe the trajectory relationship of the variables of a differential equation in the phase plane. To obtain the phase-plane portrait of a particular differential equation, a definition of the phase variables must be made. Then, by using the differential equation, the slope of the phase-plane trajectory is obtained. By computing the values of this slope in the whole plane, the phase-plane trajectory can be sketched.

The technique considered here uses a somewhat different approach. Instead of having an initial differential equation, one first sketches a trajectory corresponding to the oscillation desired. The next step is to write a differential equation which will be implemented using electronic circuits.

The interest here is on periodic oscillations. A necessary condition for this case is that the phase trajectory must be a closed contour. By choosing the form of the closed path, arbitrary waveforms can be generated. The technique is limited only by the difficulties in approximating the path in the phase plane using physical components.

Manuscript received June 21, 1977; revised January 9, 1978.
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II. Oscillations In The Phase Plane

A. Sinewave Oscillator

First, the well-known quadrature oscillator for generating sinewaves will be analyzed in the phase plane. For this case, a linear second-order differential equation is used to generate the waveforms desired. Such a differential equation is equivalent to two first-order differential equations by defining suitable phase variables.

Consider the second-order differential equation

$$\ddot{x} + \omega^2 x = 0. \tag{1}$$

By defining the phase variables $x_1 = \omega x$ and $x_2 = \dot{x}$, equation (1) becomes a set of first-order differential equations

$$\dot{x}_1 = \omega x_2 \qquad \dot{x}_2 = -\omega x_1. \tag{2}$$

From (2), the slope of the phase trajectories in the phase plane can be obtained as

$$\frac{dx_2}{dx_1} = -\frac{x_1}{x_2}. \tag{3}$$

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