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DIFFERENTIAL EQUATION MODEL OF THE FLANK WEAR

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ABSTRACT

A model of flank wear consisting of differential equations is presented. Such a model is required as the first stage towards dynamic optimization of the cutting process. Verification of the model is done by using experimental data available from the literature.

NOMENCLATURE

A, B, C_0, C_1, C_2	— constants of the differential equation
C	— the constant of Taylor's equation
n	— the speed exponent in Taylor's equation
T	— tool life
v	— cutting speed
W	— width of flank wear

INTRODUCTION

The field of optimization and economics of machining has been discussed frequently. The optimal cutting conditions depend on two functions: a mathematical model which presents the machining process and a cost criterion. The most popular criteria are the minimum cost [1, 2, 3, 4, 5, 6], the maximum production-rate [6, 7, 8], and the maximum profit-rate criterion [9, 10]. In all these approaches, particularly for an optimal cutting speed calculation, the mathematical model is unique: Taylor's equation. Since this equation is an algebraic one, using it as the mathematical model leads to a static solution of the problem, namely working with a constant cutting speed.

However, it is not necessarily true that working with a constant optimal speed will ensure the best results. It is known from the optimum control theory that the

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optimal solution results usually in a time-variable control input. In the metal cutting case applying a variable cutting speed requires expensive control equipment and therefore a discrete solution, by which each workpiece is machined with a different cutting speed, can be considered as well. In order to apply either of these solutions the differential equations which describe the process, must be known.

Some works have been done to develop mathematical models describing the wear growth of the tool. There are simple models [11] on the one hand and more comprehensive models of flank-wear [12, 13] and crater wear [14] on the other hand. However, none of these models consist of differential equations.

A comprehensive mathematical model for steel turning, which consists of differential equations, has been presented in [15]. This model shows the relation between the process parameters (cutting speed, feed, forces, temperature, etc.) and the width of the tool wear-land, W. Nevertheless, the model is too complicated and contains a great number of indirectly measured parameters. However, its time response can be approximated by the equation

$$W(t) = B - A + Ae^{t/\tau_1} - Be^{-t/\tau_2} \tag{1}$$

where A and B depend on the tool and workpiece material and show only small dependence upon the cutting conditions (speed and feed). They are assumed here to be independent of the cutting speed. The parameters τ_1 and τ_2 are strongly dependent on the cutting conditions, and the relationship $t < \tau_1$ holds during the entire machining interval. For example, it has been found in [15] that a typical value of τ_1 is about 70 min, and of τ_2 is about 5 min.

It is well known that the wear characteristic comprises three stages:

(1) a short initial stage with high wear rate

(2) a near-linear stage

(3) a final high-rate stage, representing the tool failure

Eq. (1) describes mathematically the first and second stages of the wear. Substituting the exponential expansion

$$Ae^{t/\tau_1} - A = A\left[(t/\tau_1) + \frac{(t/\tau_1)^2}{2!} + \frac{(t/\tau_1)^3}{3!} + \dots\right]$$

into Eq. (1), and bearing in mind that τ_1 is relatively large, we see that $B[1 - \exp(-t/\tau_2)]$ represents the initial stage, while a good approximation of the near-linear stage is

$$W(t) \simeq B + (A/\tau_1)t$$

This shows the validity of Eq. (1) as a wear model.

A reasonable interpretation of Eq. (1) is that the process obeys to a second-order differential equation. In this presentation, this equation will be obtained and tested by using wear curves from the available literature.

Obtaining a differential equation from a given closed form solution, i. e., Eq. (1), looks at the outset as an unusual approach. However, the reader should notice that Eq. (1) is valid only for the initial condition $W(0) = 0$ and a constant cutting speed. A tool wear following a change in the cutting speed at time t_1, will be found by solving the differential equation subject to the initial conditions $W(t_1)$ and $\dot{W}(t_1)$.

THE DIFFERENTIAL EQUATION

A differential equation which has the time response of Eq. (1) is the following:

$$\tau_1\tau_2\ddot{W}+(\tau_1-\tau_2)\dot{W}-W=A-B \tag{2}$$

with the initial conditions:

$$W(0)=0; \qquad \dot{W}(0)=A/\tau_1+B/\tau_2 \tag{3}$$

Eq. (2) is valid in the range $T\geq t\geq 0$, where T is the tool life in minutes. The parameters τ_1 and τ_2 are functions of the cutting speed, v. In order to find simple relationships for $\tau_1(v)$ and $\tau_2(v)$, the following assumption is made:

$$\tau_1 \gg T \gg \tau_2 \tag{4}$$

As will be shown later, assumption (4) holds for practical cutting conditions. Eq. (1) can be approximated at $t=T$ by:

$$W_f=W(T)=B-A+Ae^{T/\tau_1} \tag{5}$$

By using the well-known Taylor's equation

$$Tv^n=C \qquad n>1 \tag{6}$$

the following relationship can be obtained

$$\tau_1 \ln\left(\frac{W_f-B+A}{A}\right)=\frac{C}{v^n} \tag{7}$$

Assuming that A and B are not functions of v, yields

$$\tau_1=C_1/v^n \tag{8}$$

The value of τ_2 is found through low speed cutting experiments. Eq. (1) can be be approximated for small v and very small t by

$$W(t)\simeq(B/\tau_2)t \tag{9}$$

It has been found in [15] that for small t, the wear is proportional to v, and therefore

$$\tau_2=C_2/v \tag{10}$$

Substituting Eqs. (8) and (10) into (2), yields

$$\frac{C_1C_2}{v^{n+1}}\ddot{W}+\left(\frac{C_1}{v^n}-\frac{C_2}{v}\right)\dot{W}-W=C_0 \tag{11}$$

where $C_0=A-B$. C_0, C_1, C_2, and n are constants depending on the feed and depth of the cut as well as on the tool and workpiece materials.

Eq. (11) can be rewritten more conveniently as

$$C_1C_2\ddot{W}+(C_1-C_2v^{n-1})v\dot{W}-v^{n+1}W=C_0v^{n+1} \tag{12}$$

Eq. (12) is the dynamic model which describes the flank wear process of the cutting tool. The equation is a nonlinear one (the input variable is v) and has an unstable solution. Some simplification of Eq. (12) is permitted by using the relationship $C_1 \ll C_2 v^{n-1}$, which practically holds.

The general solution of Eq. (12) can be obtained for a constant v. The obtained response can then be used, for example, when applying an optimal discrete speed policy, namely, cutting of each workpiece with a different speed whose computation is based on the actual wear and wear-rate.

Defining:

$$U = v^{n-1}$$
$$E_1 = \exp(tvU/C_1)$$
$$E_2 = \exp(-tv/C_2)$$
$$F = 1/(C_1 + C_2 U)$$

The solution of Eq. (12) is given by the following equation:

$$W(t) = F(C_1E_1 + C_2UE_2)\,W_0 + C_1C_2(F/v)(E_1 - E_2)\dot{W}_0 + \\ + C_0[-1 + F(C_1E_1 + C_2UE_2)] \qquad (13)$$

W_0 and $\dot{W}_0$ are the given initial conditions. If the cutting speed is changed during $t \in [0, T]$, the initial conditions of the next period are the final conditions of the previous one. The final value of $\dot{W}$ is obtained from its corresponding equation; the latter is derived by differentiating of Eq. (13)

$$\dot{W}(t) = F(E_1 - E_2)vUW_0 + F(C_2UE_1 + C_1E_2)\dot{W}_0 + F(E_1 - E_2)vUC_0 \qquad (14)$$

Eq. (13) describes the general wear curve, while Eq. (1) is a particular case of Eq. (13) and can be obtained by substitution of Eq. (3) into the latter (bearing in mind that $C_0 = A - B$).

PRACTICAL EXAMPLES

The values of the various constants must be found from practical wear curves which were obtained for two different cutting speeds at least. The results of such

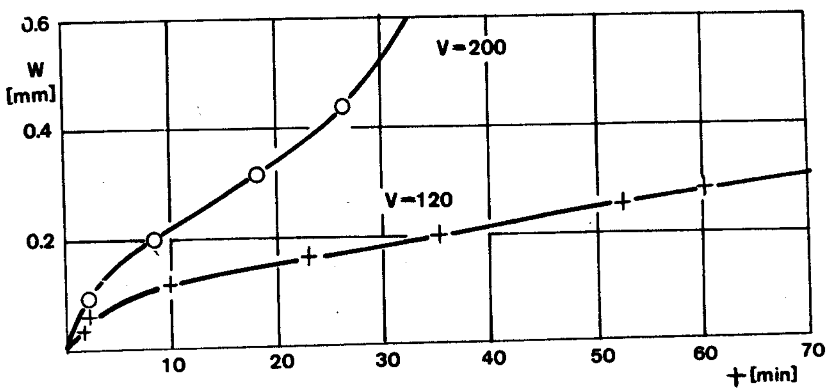

Fig. 1

a one have been presented in [16] and were copied in Figure 1. The steel is SAE 1045, the tool is P-25, the feed 0.14 mm/rev, and the depth of cut 1.5 mm. Curves produced by Eq. (12) were fitted to the measured-points and the following values have been calculated for the constants:

$$n = 2.67 \qquad C_0 = 2.605 \text{ mm}$$
$$A = 2.7 \text{ mm} \qquad C_1 = 3 \times 10^8$$
$$B = 0.095 \text{ mm} \qquad C_2 = 400 \text{ m}$$

The value of C in Taylor's equation, Eq. 6) has been calculated for $W_f = 0.4$ and $v = 200$ m/min ($T = 23$ min) and is $C = 32 \times 10^6$. For this speed $\tau_1 = 215$ min and $\tau_2 = 2$ min, which proves assumption (4).

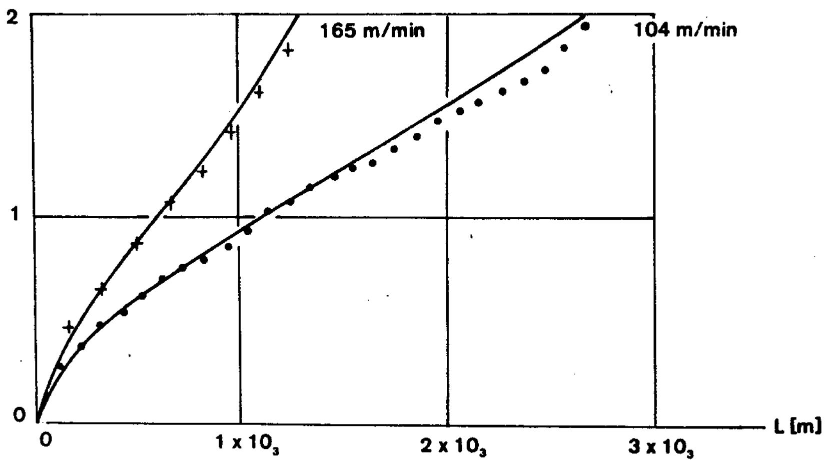

Fig. 2

As a second practical example, consider the results of machining AISI 4340 steel with CX(AA) carbide tool, which were presented in [17] and copied in Figure 2. Again, curves described by Eq. (11) were fitted and the constants were calculated:

$$n = 1.70 \qquad C_0 = 5.47 \text{ mm}$$
$$A = 5.72 \text{ mm} \qquad C_1 = 2.8 \times 10^7$$
$$B = 0.25 \text{ mm} \qquad C_2 = 100 \text{ m}$$

As the last example, the data presented in [18] have been used. The workpiece steel is AISI 1045, the tool is P-10, the feed 0.0104 ipr (0.264 mm/rev), and the depth of cut 0.03 inch (0.752 mm). The data were obtained for four different cutting speeds: 400, 600, 800, and 1000 fpm. The values of the various constants of the differential equation are as follow :

$$n = 3.0 \qquad C_0 = 0.998 \text{ inch } (2.03 \text{ mm})$$
$$A = 0.1 \text{ inch } (2.54 \text{ mm}) \qquad C_1 = 1.8 \times 10^{10} \ (5.1 \times 10^8)$$
$$B = 0.002 \text{ inch } (0.51 \text{ mm}) \qquad C_2 = 250 \text{ ft.} \ (76 \text{ m})$$

The solution of the differential equation with these constants is presented in Figure 3. Notice that in the metric system speeds are measured in m/min and wear in mm.

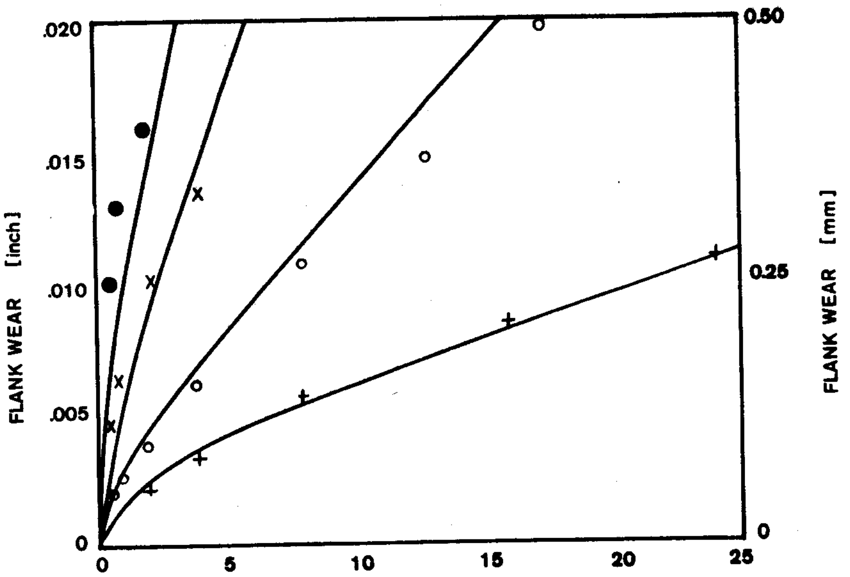

Fig. 3

CONCLUSION

A mathematical model of the flank wear which consists of a second-order differential equation has been suggested. The model permits prediction of tool life in the course of dynamic optimization of the cutting process. The model has shown a good adaptation with practical data, but which consists solely of cases with constant cutting speed. Thus, further tests, in which the cutting speed will be changed during the cutting operation, are required for a complete verification of this model.

REFERENCES

[1] Brewer, R. C.: On the economics of the basic turning operation. *Trans. ASME, J. Eng. Ind.*, Vol. 80, 1958, pp. 1479—1489

[2] Brewer, R. C. and Rueda, R.: A simplified approach to the optimum selection of machining parameters. *Eng. Digest*, Vol. 24, 1964, pp. 133—138

[3] Radford, J. D. and Richardson, D. B.: Optimization of parameters in turning. *Prod. Engr.*, Vol. 49, No. 5, May 1970, pp. 197—201

[4] Petropoulas, P. B.: Optimal selection of machining rate variables by geometric programming. *Int. J. Prod. Res.*, Vol. 11, No. 4, October 1973, pp. 305—314

[5] Ramaswamy, V. K. and Lambert, B. K.: Cost optimization for a single pass turning operation including inventory and penalty costs. *Int. J. Prod. Res*, Vol. 12, No. 3, May 1974, pp. 331—344

[6] Field, M., Zlatin, N., Williams, R., and Kronenberg, M.: Computerized determination and analysis of cost and production rates for machining operations. *Trans. ASME, J. Eng. Ind.*, Vol. 90, No. 3, August 1968, pp. 455—466

[7] Gilberg, W. W.: Economics of machining. *Machining — Theory and Practice*, American Soc. for Metals, 1950, pp. 465—485

[8] Okushima, K. and Hitomi, K.: A study of economical machining — an analysis of the maximum-profit cutting speed. *Int. J. Prod. Res.*, Vol. 3, 1964, pp. 73—78

[9] Armarego, E. J. A. and Russell, J. K.: Maximum profit-rate as a criterion for the selection of machining conditions. *Int. J. Mach. Tool Des. Res.*, Vol. 6, No. 1, March 1966, pp. 15—23

[10] Wu, S. M. and Ermer, D. S.: Maximum profit as the criterion in the determination of the optimum cutting conditions. *Trans. ASME, J. Eng. Ind.*, Vol. 88, No. 4, November 1966, pp. 435—442

[11] Müller, E.: Der Verschleiss von Hartmetall Werkzeugen und seine Kurzzeitige Ermittlung. Ph. D. Thesis, Mechanical Engineering, ETH, Zürich, 1962

[12] Depiereux, W. R.: Die Ermittlung optimaler Schnittbedingungen, insbesondere im Hinblick auf die wirtschaftliche Nutzung numerisch gesteuerter Werkzeugmaschinen. D. Sc. Thesis, Mechanical Engineering, T. H. Aachen, December 1969

[13] Bhattacharyga, A., Ghosi, A., and Ham, I.: Analysis of tool wear. Part II: Applications of flank wear models. *Trans. ASME, J. Eng. Ind.*, Vol. 92, No. 1, February 1970, pp. 109—114

[14] Lee, Y.: Theoretical model of crater wear. *Trans. ASME, J. Eng. Ind.*, Vol. 93, No. 4, November 1971, pp. 1051—1956

[15] Koren, Y. and Lenz, E.: Mathematical model of the flank-wear while turning steel with carbide tools. *CIRP Proc. Mfg. Sys.*, Vol. 1, No. 2, 1972

[16] Koren, Y.: Dynamic and static optimization of the cutting process. *Proc. First NAMR Conf.*, Hamilton, Canada, May 1973, pp. 67—93

[17] Friedman, M.: Flank wear mechanism of carbide tools. *CIRP Annals*, Vol. 15, 1967, pp. 216—221

[18] Kahng, C. H. and Patterson, G. A.: Investigation of the optimum tool life under varied cutting conditions. *Proc. 3rd NAMR Conf.*, Pittsburgh, 1975, pp. 346—360