ESTIMATION OF ABSOLUTE SPATIAL POSITION OF MOBILE SYSTEMS BY HYBRID OPTO-ELECTRONIC PROCESSOR

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ABSTRACT

A mobile system with a hybrid opto-electronic processor has been studied. The position estimation is based on analysis of landmarks as being detected by a TV-camera attached to the mobile system. The difference between the known real shape of the landmark and its image provides the information necessary for determining the relative position of the mobile system to the landmark. The parameters of the landmark image are extracted at high speed using optical processor that performs optical Hough Transform, while the coordinates of the mobile system are computed from these parameters in a digital coprocessor using fast and simple algorithms. Furthermore, different sources of errors have been analyzed, and algorithms to improve the performance of the mobile system have been developed and evaluated by computer simulation.

1. INTRODUCTION

Navigation of mobile systems (e.g., mobile robot guidance [1], or automated guided vehicle navigation) relies on fast and accurate processing of dynamic imagery [2] needed for extraction of information on coordinates, range, relative speed, and in-motion object identification. The desired navigation system should be flexible, operate at high speed with moderate computing power, and have small or no impact on the environment. Various navigation systems have been developed, e.g., wire-guided system [3], computer vision based system [4] , and beacon based system. The existing navigation systems have certain limitations: they require installation of sources or sensors in the environment, they are inflexible and slow in operation. To overcome these limitations a landmark navigated mobile system with a hybrid opto-electronic processor has been studied. This system combines the optical image processing power with the power of digital computer to reach a good balance of various desired characteristics mentioned above. This approach can be useful in solving a wide variety of navigation problems for mobile robots in different applications such as maintenance of nuclear power plants, waste management, and assembly of space stations.

In the next section, we will discuss the operational principles of the mobile system and describe the hybrid opto-electronic processing approach. In Section 3 we will discuss the positioning errors and in Section 4 we will develop a number of algorithms that allow to improve the performances of the mobile system. Finally, in Sections 5 and 6 we will provide computer simulation results and final conclusions, respectively.

2. MOBILE SYSTEM WITH A HYBRID OPTO-ELECTRONIC PROCESSOR

In the following we will first introduce the navigation algorithm based on landmarks and describe its implementation using the hybrid opto-electronic processor.

2.1. Operational Principles

The navigation algorithm is based on analysis of known landmarks (i.e., parametric curves: lines, circles, etc.) that are artificially introduced or naturally existing in the environment. The image of the landmark is detected by a TV-camera mounted on the mobile system. The shape of the landmark's image depends on the relative orientation of the TV-camera and the landmark. In this paper we deal with a 2-D situation in which the axis of the camera lens lies in a plane perpendicular to the plane of the landmark and the center of the camera lens is at the same height as the center of the land-

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mark. In the 2-D case we need only two coordinates to determine the relative position of the mobile system with respect to the landmark (see Fig. 1). For example, when a circle is used as a landmark, the TV image will be an ellipse. The four parameters of the ellipse (i.e., two axes and the two coordinates of the center) will provide the information on the relative position of the mobile system and the landmark. Since the coordinates of the landmark in the world coordinate system are known, we can easily determine the world coordinates of the mobile system.

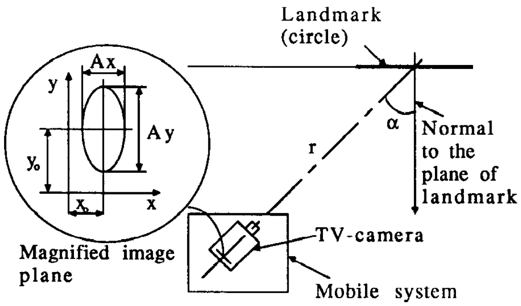


Fig. 1 Description of the relative spatial position of the mobile system and the landmark.

The relative coordinates (see Fig. 1) of the mobile system to a circular landmark are determined by

$$r^2=R^2\,\frac{f^2+A_y^2+\sqrt{(f^2+A_y^2)^2-4f^2A_y^2}}{2A_y^2} \tag{1a}$$

$$\cos\alpha=\frac{f\,A_x\,R}{r\,A_y^2}\ , \tag{1b}$$

where r and α are the polar coordinates of the mobile system with respect to the landmark, A_x and A_y are the axes of the ellipse in the image plane, R is the radius of the circular landmark and f is the focal length of the TV-camera lens.

The accuracy of the determined relative coordinates will depend on the sensitivity that is defined as the ratio of the change in the parameters of the image to the change in the mobile system position. The sensitivity depends on the relative position of the mobile system and the landmark, as it is shown in Fig. 2. At a constant angle of view (i.e., α=const) the sensitivity decreases as the distance r increases. When the distance r is held constant and the angle of view is changed, the sensitivity varies

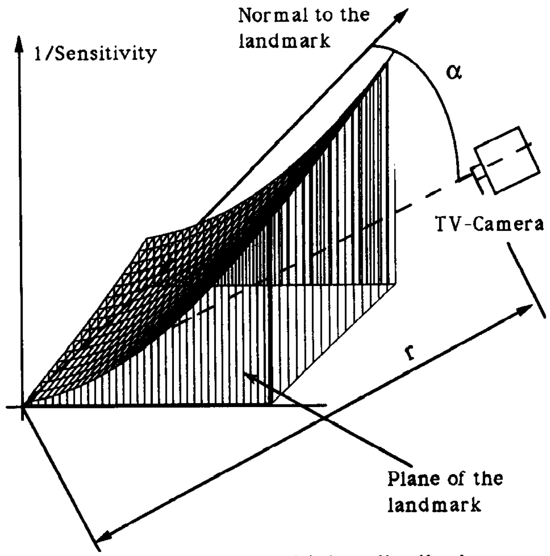


Fig. 2 Plot of the sensitivity distribution as a function of the position of the mobile system navigated with a circular landmark.

reaching its maximum value at α = 45°. The effect of the sensitivity on the positioning accuracy of the mobile system will be further discussed in Section 3.

2.2 Hybrid Opto-Electronic Processor

The block diagram of the hybrid opto-electronic processor is shown in Fig. 3. The optical processor is used to perform the time consuming operations and transformations at high speed. The image of the landmark (e.g., ellipse) is detected by a TV-camera, and introduced into the optical processor via the electronic-to-optical interface (i.e., liquid crystal display device). The parameters of the ellipse are determined at very high speed by comput-

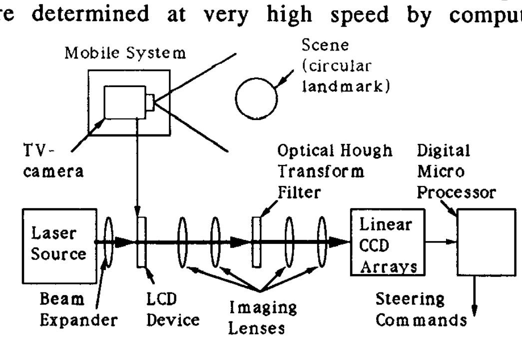


Fig. 3 Block diagram of the hybrid opto-electronic processor.

ing the Hough transform optically. The output of the optical processor (i.e., Hough transform parameter domain) is detected by the CCD-arrays and is introduced to the digital electronic microprocessor. The microprocessor is used to analyze this information, determine the relative coordinates, and provide control signals for navigation of the mobile system.

2.2.1 Detection of Parametric Curves Using Optical Hough Transform Hough Transform (HT) is a space variant transform which maps the input image plane to a parameter domain plane [5]. To detect the parameters of a straight line in normal parameterization each point in the image plane (x,y) is transformed into a sinusoidal curve in the parameter domain according to

$$\rho = x\cos\theta + y\sin\theta \quad , \qquad (2)$$

where (θ, ρ) are the coordinates of the HT parameter domain.

The amplitude distribution of the light in the parameter domain, $F(\theta,\rho)$ is related to the amplitude distribution of the light in the input plane, $f(x,y)$ according to

$$F(\theta,\rho) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy \quad , (3)$$

where $\delta(\rho - x\cos\theta - y\sin\theta)$ is the Point Spread Function (PSF) that corresponds to an input point (x,y) and the output sinusoidal curve $\rho = x\cos\theta + y\sin\theta$.

An ellipse in the image plane (x,y) is described by parametric equations

$$\begin{aligned} x &= x_0 + A_x\cos\beta \\ y &= y_0 + A_y\sin\beta \quad , \end{aligned} \qquad (4)$$

where A_x and A_y are the two axes and x_0 and y_0 are the coordinates of the center of the ellipse, and β is a parameter. Here A_x, A_y, x_0 and y_0 are the four parameters that characterize the ellipse. The HT of the input ellipse image expressed by Eq. (4) yields in the parameter domain an image that is described by the sinusoidal envelope as shown in Fig. 4.

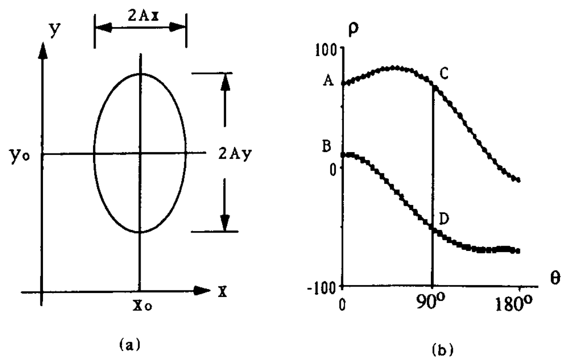


Fig. 4 Extraction of the parameters of an ellipse using Hough transform: (a) input ellipse image; (b) output image resulting from computing the Hough transform envelope, using a HT filter with impulse response for detection of a straight line in normal parameterization.

The ellipse parameters can be extracted from the amplitude distribution of light in the HT domain at $\theta = 0°$ and $\theta = 90°$ [5] (see Fig. 4):

$$\begin{aligned} F(0°,\rho) &= \int_0^{2\pi} \delta[\rho - (A_x\cos\beta + x_0)]d\beta \\ F(90°,\rho) &= \int_0^{2\pi} \delta[\rho - (A_y\sin\beta + y_0)]d\beta \quad . \end{aligned} \qquad (5)$$

The coordinates $(0°, x_0+A_x)$ and $(0°, x_0-A_x)$ of points A and B respectively (see Fig. 4) are detected and used to extract the parameters of the ellipse x_0 and A_x. The coordinates $(90°, y_0+A_y)$ and $(90°, y_0-A_y)$ of points C and D respectively provide the additional two parameters y_0 and A_y.

To expand the 2-D situation to the more general 3-D case we have to be capable of determining one more parameter that will correspond to the rotation of the ellipse. This can be accomplished by analysing the Hough transform plane along an additional line θ=const. This additional measurement will allow us to determine the phase of the envelope (see Fig. 4) providing the information on the fifth parameter, i.e., the rotation of the ellipse.

It should be noted that the position of the mobile system can be determined with high precision if there is no errors associated with the used hardware. However, practical situations involve different errors which we will analyze in the following.

3. ERROR ANALYSIS

The navigation of a mobile system employing a circular landmark involves a number of different errors due to the technological limitations. For example, the limited resolution of the TV-camera and the CCD arrays will affect the measurement accuracy, while the imperfections of the mechanical system will cause deviations of the mobile system from the designated position.

3.1 Measurement Errors

The measurement error is mainly determined by the digitization error that is affected by the sensitivity function (see Fig. 2). The digitization error occurs due to the limited resolution of the TV-camera. The minimum relative measurement error due to digitization can be determined from the size of the detected image (e.g., for a TV-camera that allows to resolve images of size 512x512 pixels, this error is on the order of 1/512=0.002). Note that if the image does not occupy the full TV-frame the relative measurement error is larger.

The minimum relative measurement error is also affected by the sensitivity function (see Fig. 2) which depends on the relative coordinates of the mobile system and the landmark (i.e., the distance r and the angle of view α). When the mobile system under a certain orientation moves away from the landmark, the relative measurement error increases due to the demagnification of the imaging system (i.e., the measurement error for an image that does not occupy full TV-frame is increased due to digitization). However, if we keep a fixed distance between the mobile system and the landmark, but change the angle of view, the relative measurement error will vary. For two extreme cases when the camera is oriented at very small angles $\alpha=0°$ (see Fig. 1) and at very large angles $\alpha=90°$, the relative measurement errors are mostly determined by the sensitivity and digitization errors respectively.

To improve the measurement accuracy we will establish a statistical model for the digitization error by assuming the ellipse parameters to be random variables that possess normal distribution. The probability model will provide reliability measure for the measurements and will allow to employ statistical techniques to improve the navigation of the mobile system.

3.2 Errors of the Mobile System

Mobile systems are suffering from different imperfections that could also introduce a number of positioning errors (e.g., for wheeled mobile robot, the diameter of the tires can be different; there can be misalignment in the two wheels; the floor conditions may be different for different wheels, etc.). In general, the error of the mobile systems is random and it will be very difficult to establish its model. However, under simplifying assumptions such a random process model can be established, especially if there is a priori knowledge on the environment and the mobile system.

3.3 Combined Error

We assume that the measurement error and the mobile system error are two independent random variables. Usually we can treat them separately by first correcting for the systematic error of the mobile system , and then applying an appropriate algorithm that will increase the positioning accuracy and reduce the effect of the measurement errors. In the following we will be concerned with the corrections for the measurement errors only.

4. ALGORITHMS FOR IMPROVED PERFORMANCE

In order to increase the robustness and positioning accuracy of the mobile system we have developed algorithms based on a weighted average method [6] :

$$\overline{X}^{(n)} = \frac{\sum_{i=1}^{n} W_i X_i^{(n)}}{\sum_{i=1}^{n} W_i} , \qquad (6a)$$

where $\overline{X}^{(n)}$ is the new estimate of the current position at the n-th point, $X_i^{(n)}$ is the prediction for the n-th point obtained from the i-th measurement according to

$$X_i^{(n)} = X_i^{(i)} + \sum_{j=i}^{n-1} \Delta X_j^{(j+1)} , \qquad (6b)$$

where $X_i^{(i)}$ is the measurement at the i-th point and $\Delta X_j^{(j+1)}$ is the increment used to move the mobile system from the j-th point to the (j+1)-th point, and W_i is the weight assigned to the value $X_i^{(n)}$.

In our approach the prediction of the current position of the mobile system is computed based on the measurement information from both the previous and the current points. In the following section we develop three different algorithms which will allow to fuse this information by providing the appropriate weighting coefficients.

4.1 Algorithm with Weights Based on Sensitivity

The sensitivity of measuring the parameters of the landmark depends on the mobile system's location as shown in Fig. 2. In general, the accuracy of the measurement increases with the increase in sensitivity, i.e., the measurement obtained in the high sensitivity region is more reliable, and should be assigned a larger weight. Therefore, for simplicity it would be natural to use the sensitivity function as a measure of the weights. The sensitivities $W_\alpha=\partial A_x/\partial\alpha$ and $W_r=\partial A_y/\partial r$ can be determined from Eq. (1) rewritten in the following form:

$$A_x = \frac{f R r \cos\alpha}{r^2-R^2\sin^2\alpha}$$
$$A_y = \frac{f R}{\sqrt{r^2-R^2\sin^2\alpha}} \qquad (7)$$

Using these sensitivities as the weights in Eq. (6a) we obtain the location of the mobile system using the following equations:

$$\bar{r}^{(n)} = \frac{\sum_{i=1}^{n} [(W_r)_i \, r_i^{(n)}]}{\sum_{i=1}^{n} (W_r)_i}$$
$$\bar{\alpha}^{(n)} = \frac{\sum_{i=1}^{n} [(W_\alpha)_i \, \alpha_i^{(n)}]}{\sum_{i=1}^{n} (W_\alpha)_i} , \qquad (8)$$

where $\bar{r}^{(n)}$ and $\bar{\alpha}^{(n)}$ are the distance and the orientation of the mobile system respectively, $(W_r)_i$ and $(W_\alpha)_i$ are the sensitivity functions determined from Eq. (7) and evaluated at the i-th point, and $r_i^{(n)}$ and $\alpha_i^{(n)}$ are the predictions for the coordinates at the n-th point which are defined by equations similar to Eq. (6b).

4.2 Algorithms Based on Optimal Weight Selection

The algorithm described by Eq. (8) is simple but it is not based on any optimization procedure. In the following we will develop algorithms for optimal weight selection, but first we will describe a reliability measure of the measurement.

4.2.1 Description of the Reliability of the Measurement

The reliability measure is a function of the digitization error and the sensitivity. The errors in determining the coordinates of the mobile system can be approximated by the digitization error and the sensitivity according to

$$\Delta r = \frac{\partial r}{\partial A_x}\Delta A_x + \frac{\partial r}{\partial A_y}\Delta A_y$$
$$\Delta\alpha = \frac{\partial \alpha}{\partial A_x}\Delta A_x + \frac{\partial \alpha}{\partial A_y}\Delta A_y , \qquad (9)$$

where ΔA_x and ΔA_y are the measurement errors of the two axes of the ellipse, and Δr and $\Delta\alpha$ are the errors in determining the distance and the orientation respectively.

Assuming A_x and A_y to be normally distributed independent random variables, we can relate the variances of the coordinates to the variances of A_x and A_y:

$$\sigma_r^2 = \left(\frac{\partial r}{\partial A_x}\right)^2 \sigma_{Ax}^2 + \left(\frac{\partial r}{\partial A_y}\right)^2 \sigma_{Ay}^2$$
$$\sigma_\alpha^2 = \left(\frac{\partial \alpha}{\partial A_x}\right)^2 \sigma_{Ax}^2 + \left(\frac{\partial \alpha}{\partial A_y}\right)^2 \sigma_{Ay}^2 , \qquad (10)$$

where σ_{Ax} and σ_{Ay} are the variances of the two axes of the ellipse, σ_r and σ_α are the variances of the coordinates of the mobile system. The values of the coordinate variances, σ_r and σ_α, can be used as measures of reliability.

4.2.2 Minimizing the Variance of the New Estimate

The second algorithm is based on minimizing the variance of the new estimate. Assuming that $X_i^{(n)}$ in Eq. (6a) are independent random variables we can determine the variance of every new estimate as

$$\sigma_{\bar{r}_n}^2 = \frac{\sum_{i=1}^{n} (W_r)_i^2 \sigma_{r_i}^2}{[\sum_{i=1}^{n} (W_r)_i]^2} , \qquad (11)$$

where σ_{r_i} is the variance of the distance coordinate at the i-th point and $\sigma_{\bar{r}_n}$ is the variance of the new estimate of the distance at the n-th point.

In order to minimize the variance we must satisfy the relation

$$\frac{\partial \sigma_{\bar{r}_n}^2}{\partial (W_r)_i} = 0 . \qquad (12)$$

Substituting Eq. (11) into Eq. (12) and solving the resultant equation we obtain the optimum weights:

$$(W_r)_i \propto 1/\sigma_{r_i}^2 . \qquad (13)$$

Similar relation can be obtained for the second coordinate(i.e., angle of view α).

4.2.3 Minimizing the Error at the Target Point

The third algorithm is based on minimizing the error at the target point. The error at the target point can be expressed by

$$J = (\bar{X}^{(n)} - X_t^{(n)})^2 , \qquad (14)$$

where $\bar{X}^{(n)}$ is the estimate for the target point given by Eq. (6) and $X_t^{(n)}$ is the coordinate of the target point. To minimize the error we have to satisfy the following equation,

$$\frac{\partial J}{\partial W_i} = 0 , \quad i=1,\cdots, n-1. \qquad (15)$$

Substituting Eq. (14) and Eq. (6a) into Eq. (15) we obtain:

$$\sum_{i=1}^{n} W_i(X_i - X_j) = 0 ,$$

where j=1,2,...,n-1. Assuming a certain value for the n-th weight, W_n we can rewrite the last equation as

$$\begin{bmatrix} 0 & X_2-X_1 & X_3-X_1 \cdots X_{n-1}-X_1 \\ X_1-X_2 & 0 & X_3-X_2 \cdots X_{n-1}-X_2 \\ & \cdots\cdots & \\ X_1-X_{n-1} & X_2-X_{n-1} & X_3-X_{n-1} \cdots 0 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \\ \cdots \\ W_{n-1} \end{bmatrix} = W_n \begin{bmatrix} X_1-X_n \\ X_2-X_n \\ \cdots \\ X_{n-1}-X_n \end{bmatrix} . \qquad (16)$$

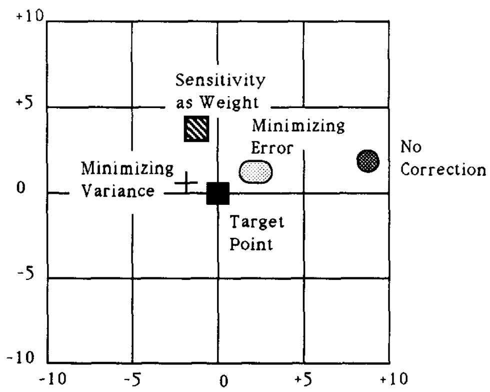
Solving this system of linear equations we find the optimal weights for minimizing the error at the target.

5. COMPUTER SIMULATION RESULTS

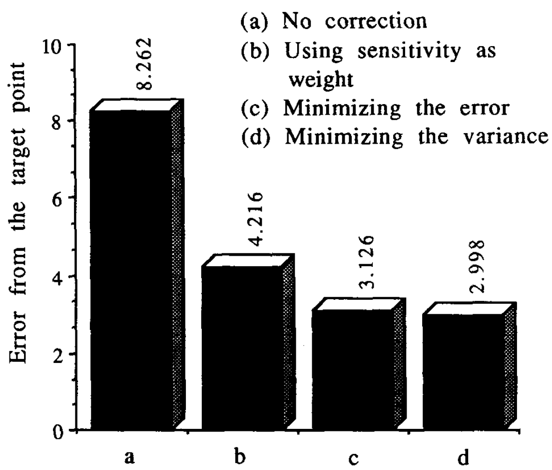
We have performed computer simulations employing different algorithms introduced in Section 4. In the simulations we have instructed the mobile system to move from one point to another along a piecewise straight line path. While executing the movement the mobile system performs measurement at points separated by fixed steps. A new estimate of the current position is obtained at each measurement point, by using the new as well as the previous measurements and weighting factors determined by the three algorithms discussed above. The data from the tracking system on board of the mobile system is also used. The mobile system plans the next straight line path according to the new estimate and the target position. The final errors at the target point are determined and compared for three different algorithms as well as for a system with no correction. These computer simulation results are shown in Figs. 5a and b. The performance of the second algorithm based on minimizing the variance is superior as compared to the performances of the algorithms based on the sensitivity and minimizing the error at the target point.

6. CONCLUSIONS

In this paper, a mobile system with a hybrid opto-electronic processor has been studied. The processor combined the power of optical image processing and digital computing to meet the real time applications and achieve high accuracy. We analyzed different error sources that affect the navigation of the mobile



(a)



(b)

Fig. 5 Computer simulation results with following parameters: coordinates of the starting point, (500,500); coordinates of the target, (0,0); coordinates of the circular landmark, (1000, 1100); incremental step size, 100; image resolution, 128x128; focal length of the TV-camera lens, f=20mm; and radius of the circular landmark, R=200mm. (a) Performances of the navigation algorithm with different error reduction criteria and (b) Comparison of the navigation errors resulting from employing different error correcting criteria.

system. To assure robust and accurate operation of the mobile system we have developed algorithms based on satisfying different criteria. The best performances in the navigation of the mobile system were obtained with the algorithm based on minimizing the variance of the new estimate.

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