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# Flank Wear Model of Cutting Tools Using Control Theory

*A model of the flank wear of cutting tools is developed by using linear control theory. The flank wear is assumed to consist of a mechanically activated and a thermally activated component. The wear process is mathematically treated as a feedback process, whereby the progressive wear raises the cutting forces and temperature thereby increasing the thermally activated wear-rate, and contributes to the mechanically activated wear. A mathematical expression for the flank wear growth is derived and shown to be consistent with experimental results. The experimental data is fitted to the wear model for calculating the mechanical wear coefficient and activation energy for the thermally activated wear. The model yielded a new tool-life equation which is valid over a wider range of speed than Taylor tool-life equation.*

## Introduction

The determination of optimal cutting conditions in machining operations requires a mathematical relationship between the tool life and machining parameters. For this purpose a Taylor equation is widely used in which the tool life is related to the cutting speed by a power function relationship. This type of relationship is based upon empirical results rather than upon a physical model of the wear process. Therefore, it is not surprising that there are many cases in which the Taylor equation does not appear to be valid [1],[1] and accelerated tool life testing based on the Taylor equation can lead to large errors in tool life prediction [2]. Better results might be obtained by directly relating the tool life to a model of the wear land growth [3-5]. Our approach along this line [5] has been to use a physical wear model which is based on a feedback relationship. The present study involves modifications of this model, yielding consequently a mathematical expression for the flank wear growth. This study also concerned with the practical testing of the model.

For the feedback flank wear model [5], it is assumed that wear occurs by two principal mechanisms: a thermally activated one and a mechanically activated one. Each of these mechanisms is presented by a separate branch in the block diagram of the model which is shown in Fig. 1. The wear-rate due to the thermally activated mechanism is temperature dependent according to the well known Arhennius

equation, which is highly nonlinear. The integrator (1/s) provides the conversion from wear-rate to wear. The mechanically activated mechanism (which is taken to be independent of temperature) is presented by a first order lag with a time constant of $\tau$, which varies inversely with the cutting speed. The total wear, $W$, at any time is the sum of the wear due to the thermally activated mechanism $W_1$ and the wear due to the mechanically activated mechanism, $W_2$:

$$W = W_1 + W_2 \tag{1}$$

As the tool wear land grows, the cutting forces increase due to sliding between the wear land and the workpiece. The power component of the cutting force can be written as [4, 5, 6]

$$F = F_o + K_3 W \tag{2}$$

where $F_o$ is the initial force for the sharp tool and $K_3$ is a constant. This increase in power force results in higher temperatures so that the thermally activated wear rate $W_1$ becomes bigger. The total effect is like that of a positive feedback loop: the wear-rate is temperature
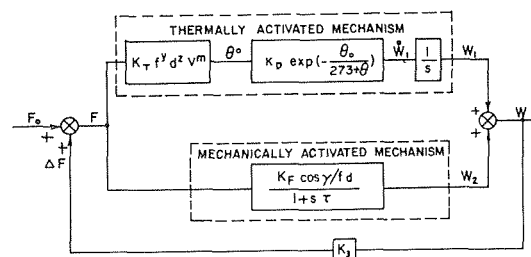


Fig. 1   The original model
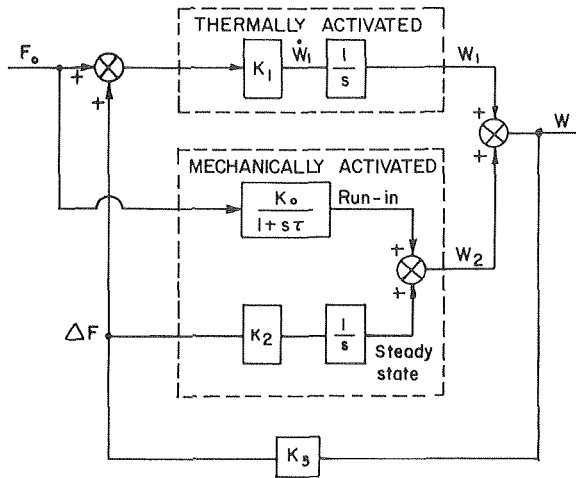
**Fig. 2  The modified model**



**Fig. 3  Block diagram of the model**

dependent and the temperature in turn is dependent on the wear [7, 8].

A drawback for practical utilization of the model shown in Fig. 1 is the enormous number of constants ($F_o$, $K_T$, $K_D$, $K_F$, $\theta_o$, $\tau$, $\gamma$, $n$, $y$, $z$) most of which cannot be evaluated directly from wear curves. In the present paper a modification of this feedback model is presented. It yields a mathematical expression for the wear which allows for evaluation of parameters from actual flank wear curves. Experimental results are fitted to the model permitting separate observations of the thermally activated and mechanically activated wear components. In addition, estimations of the activation energy, for the thermally activated wear, and the mechanical wear constant are provided. The classic Taylor tool-life equation is derived from the model for a moderate range of cutting speeds, while the model yields another tool-life equation which is valid over a wider range of speeds.

## The Model

A block diagram of the modified model is shown in Fig. 2. The parameters $K_o$, $K_3$ and the force $F_o$ are assumed to be speed independent and are dependent on the feed and depth of cut. The parameters $K_1$, $K_2$ and $\tau$ are dependent upon the cutting speed as well as on the other cutting conditions. The main modifications to the model in Fig. 1 incorporate linearization of the thermally activated wear-rate component and an improved description of the mechanically activated component. In addition, fewer constants are required for using the modified model. The linearization of the exponential term permits the employment of the Laplace transformation technique. For the mechanically activated mechanism there is an initial high "run-in" wear rate which decreases to an almost "steady-state" wear-rate. The duration of the initial run-in stage depends on the time constant $\tau$, which varies inversely with the speed. The model assumes that the mechanically activated steady-state wear has the classical linear wear behaviour and is proportional to the force acting on the flank surface and the speed. The force acting on the flank surface is proportional
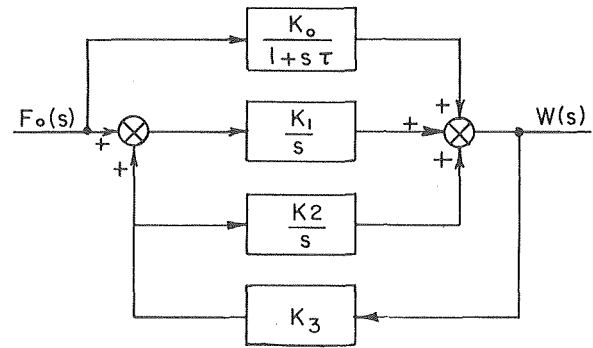
to the additional force $\Delta F$ and the parameter $K_2$ is proportional to the cutting speed. The integrator (l/s) provides the conversion from wear-rate to wear.

For mathematical convenience the model is redrawn in Fig. 3. From control theory, the Laplace-transformed wear $W(s)$ can be expressed as:

$$W(s) = \frac{K_o F_o(s)}{1 + s\tau} + \frac{K_1 F_o(s) + K_1 K_3 W(s)}{s} + \frac{K_2 K_3 W(s)}{s} \quad (3)$$

Equation (3) is solved for $W(s)$:

$$W(s) = \frac{[K_1 + s(K_o + \tau K_1)]F_o(s)}{D(s)} \quad (4)$$

where

$$D(s) = \tau s^2 + (1 - \tau K_1 K_3 - \tau K_2 K_3)s - K_1 K_3 - K_2 K_3 \quad (5)$$

The equation $D(s) = 0$ is the characteristic equation of the process. The roots of this equation are denoted by a and b such that equation (5) can be rewritten as

$$D(s) = \tau(s - a)(s + b) \quad (6)$$

where

$$a = K_1 K_3 + K_2 K_3 \quad (7)$$

$$b = 1/\tau \quad (8)$$

For constant feed and depth of cut, $F_o(s)$ is a step input, namely

$$F_o(s) = F_o/s \quad (9)$$

Substituting equations (6) and (9) into (4) yields:

$$W(s) = \frac{K_1 F_o}{s(s - a)} + \frac{K_o F_o}{D(s)} \quad (10)$$

Taking the inverse transform of equation (10) gives the time dependence of the wear land:

$$W(t) = A(e^{at} - 1) + B(1 - e^{-bt}) \quad (11)$$

where

$$A = \frac{K_1 F_o}{a} + \frac{K_o F_o}{\tau(a + b)} \tag{12}$$

$$B = \frac{K_o F_o}{\tau(a + b)} \tag{13}$$

Similarly, the Laplace-transformed wear components $W_1(s)$ and $W_2(s)$ in Fig. 2 can be expressed as:

$$W_1(s) = \frac{K_1 F_o(s) + K_1 K_3 W(s)}{s} \tag{14}$$

$$W_2(s) = \frac{K_o F_o(s)}{1 + s\tau} + \frac{K_2 K_3 W(s)}{s} \tag{15}$$

Substituting for $F_o(s)$ and $W(s)$ from equations (9) and (10) yields

$$W_1(s) = \frac{(s - K_2 K_3)K_1 F_o}{s^2(s - a)} + \frac{K_o K_1 K_3 F_o}{s D(s)} \tag{16}$$

$$W_2(s) = \frac{K_1 K_2 K_3 F_o}{s^2(s - a)} + \frac{(s - K_1 K_3)K_o F_o}{s D(s)} \tag{17}$$

Taking the inverse transforms of equations (16) and (17) gives the time dependence of the thermally and mechanically activated mechanisms, respectively

$$W_1(t) = A_1(e^{at} - 1) + B_2(e^{-bt} - 1) + Kt \tag{18}$$

$$W_2(t) = A_2(e^{at} - 1) + B_2(1 - e^{-bt}) - Kt \tag{19}$$

where

$$A_1 = \left[\frac{K_1 F_o}{a^2} + \frac{K_o F_o}{\tau a(a + b)}\right] K_3 K_1 = \frac{K_1}{K_1 + K_2} A \tag{20a}$$

$$A_2 = \frac{K_2}{K_1} A_1 = \frac{K_2}{K_1 + K_2} A \tag{20b}$$

$$B_1 = \frac{K_o K_1 K_3 F_o}{a + b} \tag{21a}$$

$$B_2 = K_o F_o - \frac{K_2}{K_1} B_1 \tag{21b}$$

$$K = \frac{K_1 K_2 K_3 F_o}{a} \tag{22}$$

Note that by combining equations (20a) with (20b) one obtains

$$A = A_1 + A_2$$

Similarly, combining equations (21a) and (21b) yields

$$B = B_2 - B_1$$

The parameters $A$ and $B$ are defined by equations (12) and (13), respectively. As a consequence the wear equation, Eq. (11), is also obtained by summing of equations (18) and (19).

## Identification of Parameters

Cutting tool wear behavior is typically characterized by a high initial wear rate followed by an almost constant "steady-state" wear rate. At some later time the wear rate may increase abruptly as the tool completely fails. An example of a wear curve plotted according to equation (11) is shown in Fig. 4 for $A = 1.5$mm, $B = 0.1$mm, $1/a = 80$min, $1/b = 2$min. It is seen that the curve describes the wear growth in the initial and steady state stages. The time duration of the initial run-in stage of wear is inversely proportional to $b$. Therefore, the value of $1/b$ is typically much smaller than the tool life. The interval usually taken as linear is exponential in the present model with a relatively large time constant $1/a$. For this region, this exponential term can be approximated by

$$e^{at} = 1 + at \tag{23}$$

Combining this with equation (11) gives:

$$W(t) = B + Aat - Be^{-bt} \tag{24}$$

The term $Be^{-bt}$ decays rapidly and becomes negligible when $t = 2/b$
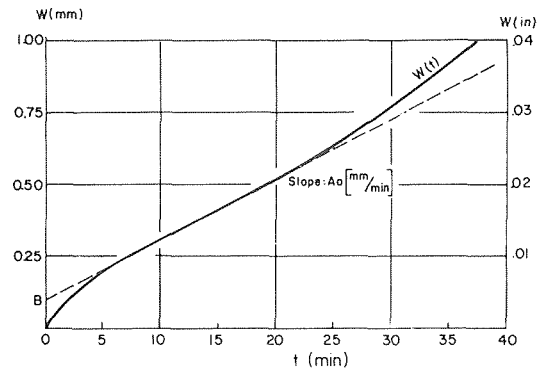


Fig. 4 Typical wear curve

at which point the near-linear interval can be considered to begin. Therefore, the linear wear can be approximated by

$$W(t) = B + Aat \tag{25}$$

Equation (25) provides for the evaluation of the parameters $B$ and $Aa$ (see Fig. 4). The slope of the linear portion is $Aa$, and the intersection of its imaginary continuation with the $W$-axis gives the value of $B$. These values of $B$ and $Aa$ are used with equation (24) at small $t$ in the run-in region to evaluate the parameter $b$.

In order to find separately the values of $A$ and $a$, the effect of the exponential term $e^{at}$ must be considered, which means that a large $t$ should be applied. The largest applicable $t$ is the tool life $T$. Equation (11) is approximated at $t = T$ by

$$W_f = W(T) = ae^{aT} - A + B \tag{26}$$

Since $W_f$ and $T$ are given, and $aA$ and $B$ were already found, the values of $A$ and $a$ can each be calculated from equation (26) by a trial-and-error method.

Considering the above discussion, it is seen that the wear process for a given condition can be characterized by four parameters $A$, $B$, $a$, and $b$—which can be evaluated from experimental wear curves. The next step is to find the cutting speed dependence of these parameters.

For typical cutting conditions the parameter $b$ is much bigger than $a$, because the time constant for the initial "run-in" wear, $1/b$, is much smaller than the time constant for the "steady-state" region, $1/a$. In addition, it will be shown from the results to be presented that $K_1 \gg K_2$, thus the parameter $a$ is approximated by

$$a \simeq K_1 K_3$$

and consequently equations (12) and (13) can be approximated by

$$A = (F_o/K_3) + K_o F_o \tag{27}$$

$$B = K_o F_o \tag{28}$$

For the purpose of this analysis the power force $F_o$ is assumed to be speed independent. The parameter $K_o$, representing the run-in stage of the mechanically activated mechanism, is speed independent as well. The parameter $K_3$ represents the feedback mechanism (see Fig. 2) and has been found to be independent of cutting speed [4, 9]. Therefore, the parameters $A$ and $B$ remain almost constant with speed, so that the speed dependence of the wear rate must enter through the parameters $a$ and $b$. The parameter $b$ affects the initial run-in stage, while the parameter $a$ affects the steady-state wear rate.

Consider first the speed dependence of the parameter $a$. From equation (7)

$$a = (K_1 + K_2)K_3 \tag{29}$$

Since $K_3$ is speed independent, the speed dependence must be related

to $K_1$ and $K_2$ which are proportional to the wear rate components of the thermally and mechanically activated mechanis, respectively. It will be shown from the results to be presented that this speed dependence can be approximated over a moderate range of speeds by

$$K_1 + K_2 = cv^n \tag{30}$$

where $c$ is a constant. Defining a new constant $C_1$

$$C_1 = 1/cK_3$$

and combining equations (29) and (30), gives

$$a = v^n/C_1 \tag{31}$$

Returning to equation (26), which can be rewritten as

$$aT = \ln[(W_f + A - B)/A] = C_o = \text{constant} \tag{32}$$

and combining it with equation (31), yields:

$$v^n T = C_o C_1 = C \tag{33}$$

which is the classic Taylor tool-life equation.

Next, consider the speed dependence of the parameter b. The parameter b is given in equation (8) as $b = 1/\tau$, and the time constant $\tau$ is inversely proportional to the cutting speed. Therefore, $b$ is proportional to the cutting speed:

$$b = v/C_2 \tag{34}$$

where $C_2$ is measured in length units.

The mathematical wear model given by equation (11) together with equations (31) and (34), provides a complete mathematical description of the wear land growth in machining:

$$W(t) = A(e^{v^n t/C_1} - 1) + B(1 - e^{-vt/C_2}) \tag{35}$$

Likewise, the contribution of the wear components $W_1(t)$ and $W_2(t)$ can be obtained from equations (18) and (19) by substituting for $a$ and $b$ from equations (31) and (34).

The parameters $A$, $B$, $C_1$, $C_2$, and $n$ depend on the feed, depth of cut, and the tool and workpiece materials. The parameters $A$, $B$, $a$, and $b$ can be determined from actual wear curves by the method that has been previously explained. The parameter n is the Taylor equation exponent. $C_1$ and $C_2$ are found through equations (31) and (34), respectively.

In order to evaluate the parameters of the model shown in Fig. 3 an additional measurement, of the power force $F_o$, is required. The feedback parameter $K_3$ might be either measured or evaluated from equations (27) and (28):

$$K_3 \simeq F_o/(A - B) \tag{36}$$

Equation (36) is based on the assumption that $K_1 \gg K_2$, which is especially valid at high speeds. The parameters of the mechanically activated run-in stage are determined from equation (28):

$$K_o = B/F_o \tag{37}$$

and from equations (8) and (34)

$$\tau = C_2/v \tag{38}$$

The parameter $K_1$ of the thermally activated mechanism is evaluated from equation (12), which can be rewritten as

$$A = K_1 F_o/a + B \tag{39}$$

and, consequently,

$$K_1 = \frac{A - B}{F_o} a \simeq \frac{v^n}{K_3 C_1} \tag{40}$$

The approximated term in equation (40) is based on (36) and therefore holds only at high speeds.

A direct evaluation of the parameter $K_2$ is more complicated since its order of magnitude is around $10^{-6}$ [mm/kg-min]. An alternative approach, based on approximation (30), is proposed here.

Combining equations (29) and (31) gives

$$\frac{C_o}{K_3(K_1 + K_2)} \simeq \frac{C_o C_1}{v^n} = \frac{C}{v^n} \tag{41}$$

The right side in equation (41) is the tool-life $T$, as given by Taylor's equation, equation (33). On the other hand, the speed dependence of the left side term in equation (41), which enters through the parameters $K_1$ and $K_2$ ($K_3$ and $C_o$ are speed independent), must be derived. Comparing the original model of Fig. 1 with the modified model shown in Fig. 2, yields the following relationship:

$$K_1 = K_1' e^{-\theta_o/(273+\theta_a)} \tag{42}$$

where

$$K_1' = K_D/K_T v^m f^v d^z/\theta_a = K_D/F_a$$

$F_a$ and $\theta_a$ are average values of the cutting force and temperature for a certain speed. $\theta_o$ is a constant depending on the activation energy and can be estimated by a method to be presented. The temperature $\theta_a$ can be evaluated by the formula [5, 6]

$$\theta_a = K_\theta v^m \tag{43}$$

Assuming that $K_\theta$ is a known constant, the speed dependence of the exponential term in equation (42) is known.

The parameter $K_2$ is the ratio between the mechanically activated steady-state wear rate and the additional force on the flank surface. So it must be proportional to speed and can be expressed by

$$K_2 = K_2' v \tag{44}$$

As a consequence, the speed dependence of the left side of equation (41) is known:

$$T = \frac{C_o/K_3}{K_1' e^{-\theta_o/273+\theta_a} + K_2' v} \tag{45}$$

The constant $C_o$ depends upon the final wear land and determined by equation (32). Choosing now two actual tool-life values for two different speeds results in two equations. Solving these equations for the two unknowns: $K_1'$ and $K_2'$, completes the evaluation of the model parameters from actual flank wear curves.

## Estimation of Wear Constants

Since the proposed model separates the thermally and the mechanically activated wear components, it allows for estimation of the activation energy on the one hand, and the mechanically activated wear coefficient on the other hand.

(a) The Activation Energy. The order of magnitude of the activation energy might be estimated from equation (41). At high cutting speeds $K_1 \gg K_2$ thus equation (41) is approximated by

$$\frac{C_o}{K_1 K_3} = \frac{C}{v^n} \tag{46}$$

Substituting for $K_1$ from equation (42) yields:

$$(CK_3 K_1'/C_o)e^{-\theta_o/(273+\theta_a)} = v^n \tag{47}$$

or

$$\ln(CK_3 K_1'/C_o) - \theta_o/(273 + \theta_a) = n \ln(v) \tag{48}$$

where $\theta_a$ is given by equation (43) and the other parameters are speed independent. Differentiating both sides of equation (48) with respect to $\ln(v)$, (in order to compare slopes in equation (47) on a logarithmic chart) gives:

$$\theta_o = \frac{n(273 + \theta_a)^2}{m\theta_a} \tag{49}$$

For $v = 200$m/min and $K_\theta = 80$, equation (43) provides a temperature estimate of 869 °C. Typical values of carbide tools are $n = 3$ and $m = 0.45$. For this numerical data equation (49) results in $\theta_o = 10000°$ C. The activation energy $U$ is calculated from

$$U = R\theta_o \tag{50}$$

where $R$ is the universal gas constant 1.98cal/mole C. Since $\theta_o = 10000$, the resulted activation energy is 20Kcal/mole. The same value was obtained for crater wear when cutting with tungsten carbide tools at low speeds [7]. This suggests that the thermally activated wear land growth mechanism at high cutting speeds may be the same as that of crater wear at low cutting speeds.

**(b) Wear Coefficient.** The rate of volume removed by a mechanically activated mechanism at the steady-state is given by [10]

$$\dot{V}_2 = K_m N v \tag{51}$$

where $N$ is the normal load on the surface and $K_m$ is the volume wear per unit load per unit sliding distance. In machining the relationship between the normal force on the flank surface and the friction force $\Delta F$ acting at the flank is constant [6, 9]:

$$\Delta F = hN$$

Since $\Delta F = K_3 W$, equation (51) becomes

$$\dot{V}_2 = (v/h)K_m K_3 W \tag{52}$$

On the other hand, the total wear volume is given by [2]

$$V = \frac{1}{2} d \tan \zeta W^2 \tag{53}$$

where $d$ is the depth (or width) of cut and $\zeta$ is the clearance angle. Differentiating of equation (53) with respect to time yields:

$$\dot{V} = d \tan \zeta W \dot{W} = d \tan \zeta W (\dot{W}_1 + \dot{W}_2) \tag{54}$$

The mechanically activated wear volume rate is given by

$$\dot{V}_2 = d \tan \zeta W \dot{W}_2 \tag{55}$$

Comparing equations (52) and (55) at the steady state gives:

$$K_m = \frac{hd \tan \zeta \dot{W}_{2s}}{vK_3} \tag{56}$$

where $\dot{W}_{2s}$ is the mechanically activated wear rate component at the steady-state. The mechanically activated wear at steady-state is derived from equation (19) with substitution of (23) for $e^{at}$ and neglecting the term $e^{-bt}$

$$W_{2s} = B_2 + (aA_2 - K)t \tag{57}$$

Differentiating of (57) and substituting the definitions for $A_2$ and $K$ from equation (20) and (22) results in:

$$\dot{W}_{2s} = \frac{K_2 K_3 K_o F_o}{\tau(a + b)} \tag{58}$$

and finally combining (58) with (13) and (44) yields
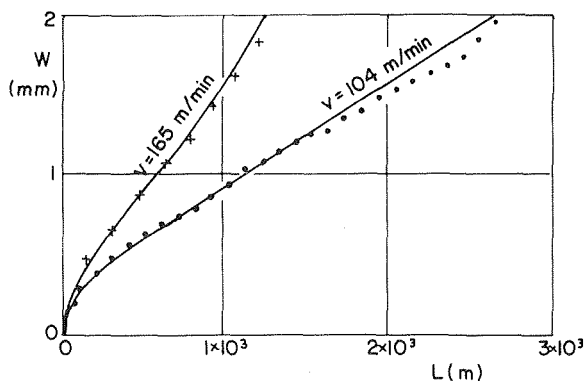
$$\dot{W}_{2s} = BK_3 K_2' v \tag{59}$$



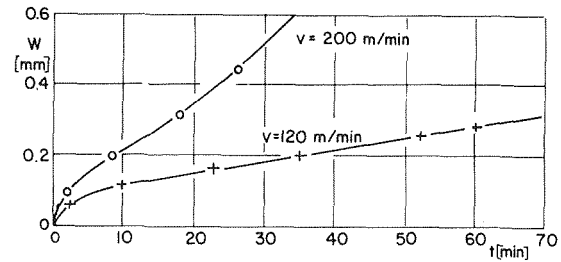Fig. 5 Measured and calculated wear curves (AISI 1045 steel, $f$ = 0.0012 ipr; d = 0.5 mm)



Fig. 6 Measured and calculated wear curves (AISI 1045 steel; $f$ = 0.14 mm/rev; $d$ = 1.5mm)

Substituting of equation (59) into (56) gives

$$K_m = hd \tan \zeta BK_2' \tag{60}$$

Mechanically activated wears are catalouged in terms of a wear coefficient $z$ defined as [2]

$$z = 3HK_m \tag{61}$$

where $H$ is the hardness of the material. Thus, an evaluation of $K_m$ provides an estimation of the wear coefficient.

## Machining Tests

In order to evaluate the parameters appearing in the wear equation, Equation (35), it is necessary to have results for at least two different cutting speeds. Consider first the results of machining AISI 4340 steel with CX(AA) carbide tool, as presented in reference [11] and shown in Fig. 5. Curves described by equation (35) were fitted and the constants obtained were:

$$n = 1.70 \qquad C_1 = 2.8 \times 10^7$$

$$A = 5.72\text{mm} \qquad C_2 = 100\text{m}$$

$$B = 0.25\text{mm}$$

As a second practical example, flank wear curves obtained by turning experiment at a constant feed (0.14mm/rev) and depth-of-cut (1.5mm) are given in Fig. 6. Results were obtained for an ISO P-20 carbide tool, turning SAE 1045 steel at 120m/min and 200m/min. The solid lines were obtained by least-square fitting correspond to equation (35) with the following constants:

$$n = 2.67 \qquad C_1 = 1.2 \times 10^8$$

$$A = 1.0\text{mm} \qquad C_2 = 400\text{m}$$

$$B = 0.095\text{mm}$$

The value of $C$ in Taylor's equation (equation (33)) for $W_f = 0.3$mm and $v = 200$m/min ($T = 16$min) is $C = 2.2 \times 10^7$. For this speed $1/a = 86$min and $1/b = 2$min, which shows that the assumption $b \gg a$ holds.

In order to evaluate the parameters of the model power force measurements were carried out as well. Fig. 7($a$) shows the relationship between the power force and the wear land. The slope of the lines is the parameter $K_3$ which was calculated by least square fitting, resulting in $K_3 = 60$kg/mm. The experimental results are somewhat surprising because of the relatively big difference between the initial forces at 120m/min (65kg) and 200m/min (55kg). The analysis is carried out with an average speed independent power force of 60kg. The mechanically activated run-in parameter is calculated from equation (37):

$$K_o = 0.095/60 = 1.58 \times 10^{-3} \text{ [mm/kg]}$$

In order to evaluate the parameters $K_1'$ and $K_2'$ a wear land of 0.3mm is taken as a tool-life criterion. The corresponding values for speeds of 120 and 200mm/min are 62.8 and 16.1min, respectively. The constant $C_o$ is determined from equation (32) for $W_f = 0.3$mm. The
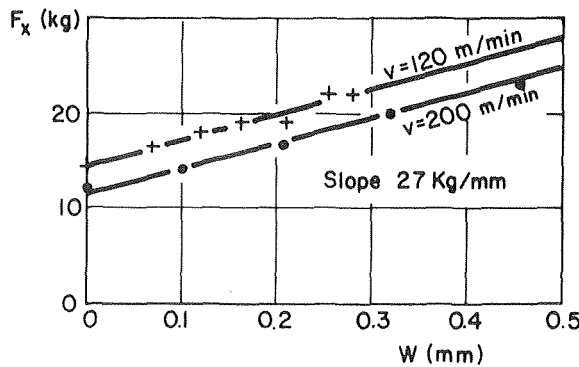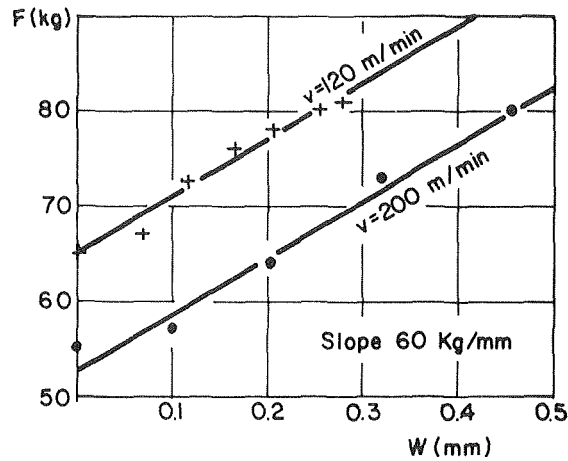
Fig. 7 Power force versus cutting time



Fig. 8 Tool life versus cutting speed

parameter $\theta_o$ was estimated for $n = 3$ as $10000°C$. Since in our case $n = 2.67$ and $\theta_o$ is proportional to $n$ (equation (49)) a value of $\theta_o = 8900°C$ is assumed. The temperature $\theta$ for the given feed (0.14mm/rev) and depth of cut (1.5mm) is approximated by [5]

$$\theta_a = 80v^{0.45} \tag{62}$$

Equations corresponding to equation (45) are derived from this data:

$$62.8 \, (9.69 \times 10^{-5} K_1' + 120 K_2') = 3.11 \times 10^{-3}$$

$$16.1 \, (40.97 \times 10^{-5} K_1' + 200 K_2') = 3.11 \times 10^{-3} \tag{63}$$

which have the solution $K_1' \simeq 0.45$mm/kg-min; $K_2' \simeq 5 \times 10^{-8}$mm/kg-m. As a consequence the tool-life equation is given by

$$T = 3.11 \times 10^{-3}/(0.45 e^{-8900/(273+\theta_a)} + 5 \times 10^{-8} v) \tag{64}$$

where $\theta_a$ is given in equation (62). The tool-life defined by equation (64) was plotted against cutting speed in a logarithmic scale (see Fig. 8).

For comparison a tool-life calculated from Taylor's equation

$$Tv^{2.67} = 2.24 \times 10^7 \tag{65}$$

was plotted on the same chart. the two curves almost coincide in the range $70 < v < 260$m/min (error less than 5 percent) which confirms the approximation given in equation (30). In the lower speed range the curve obtained by equation (64) is seen to agree with practical curves [1], while it is known that Taylor's equation is valid only over a certain range of speeds.

The coefficient of the mechanically activated wear $K_m$ can be estimated from equation (60). The normal force acting on the flank is
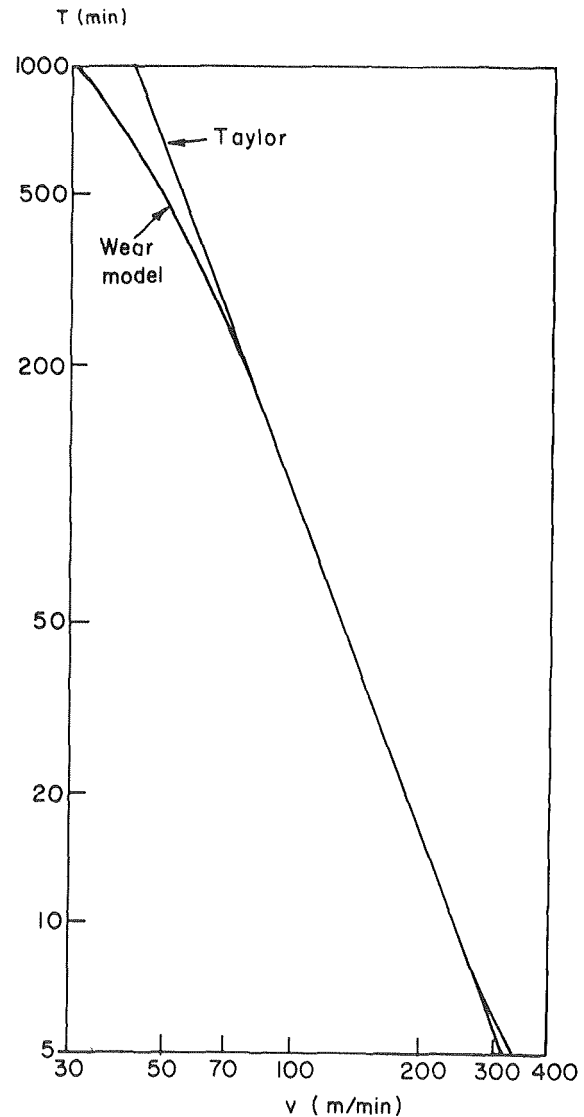
the horizontal force $F_x$ given in Fig. 7(b). Comparing Figures 7(a) and 7(b) shows that

$$\frac{dF}{dW} = \frac{60}{27} \frac{dF_x}{dW}$$

namely, $h = 2.22$. (For comparison, $h = 1.72$ has been obtained in [9] for the same tool and workpiece material, $d = 1$mm, $f = 0.2$mm/rev.) The depth of cut in this experiment was 1.5mm and the clearance angle 6 deg. The parameter $B$ and $K_2'$ are 0.095mm and 5 $\times$ $10^{-8}$m/kg-m, respectively. This data results in

$$K_m = 1.66 \times 10^{-9} \left[ \frac{\text{mm}^3}{\text{kg} - \text{m}} \right] = 1.66 \times 10^{-12} \left[ \frac{\text{mm}^2}{\text{kg}} \right] \tag{66}$$

The wear coefficient $z$ might be determined from equation (61).

The derivation of the model's parameters permits the plotting of the thermally and mechanically activated wear components against time. The required constants are calculated from equations (12), (13), (20), (21), (22), (42), (44), and (62), and are summarized in Table 1.

Fig. 9 shows the time dependence of the thermally and mechanically activated wear components for $v = 200$m/min. Curves were drawn according to equations (18) and (19) with parameters given in Table 1. We see that upon the termination of the run-in range, the wear land growth mechanism is mainly thermally activated.
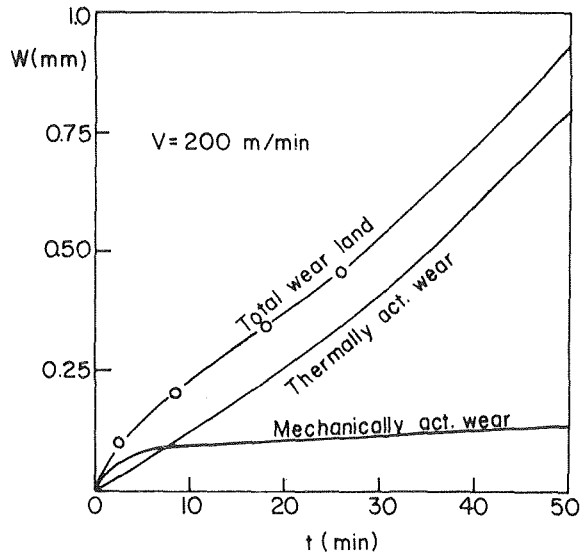
Fig. 9  Thermally and mechanically activated wear

Table 1  Parameters of the model

| Parameter | v = 120m/min | v = 200m/min | Units |
|---|---|---|---|
| $K_0$ | $1.58 \times 10^{-3}$ | $1.58 \times 10^{-3}$ | mm/kg |
| $K_1$ | $4.35 \times 10^{-5}$ | $18.4 \times 10^{-5}$ | mm/kg-min |
| $K_2$ | $0.60 \times 10^{-5}$ | $1.00 \times 10^{-5}$ | mm/kg-min |
| $K_3$ | 60 | 60 | kg/mm |
| $a$ | $2.97 \times 10^{-3}$ | $1.16 \times 10^{-2}$ | 1/min |
| $b$ | 0.3 | 0.5 | 1/min |
| $A$ | 0.974 | 1.032 | mm |
| $A_1$ | 0.846 | 0.974 | mm |
| $A_2$ | 0.128 | 0.058 | mm |
| $B$ | 0.095 | 0.094 | mm |
| $B_1$ | $8 \times 10^{-4}$ | 0.002 | mm |
| $B_2$ | 0.096 | 0.096 | mm |
| $K$ | $3.4 \times 10^{-7}$ | $6.1 \times 10^{-4}$ | mm/min |

## Conclusions

A physical model of the flank-wear, based on a feedback mechanism has been presented. Two principal mechanisms were assumed as wear causes: a thermally activated one and a mechanically activated one. The total wear occuring on the flank surface of the cutting tools is equal to the sum of the wear due to the separate effects of these mechanisms. The model yields a mathematical expression describing the wear land growth with time. Good agreement with practical data has been demonstrated.

The parameters of the model can be evaluated directly from experimental wear curves with an additional measurement of the cutting force. As a consequence, the relative weight of the thermally and mechanically activated wear components is known.

The model yields the tool life as a function of the cutting speed, and provides a formula for the tool life (equation (45)) which is not based on Taylor's equation. In addition the experimental results were fitted to the model providing estimations for the activation energy and the coefficient of the mechanically activated wear.

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