Now, proceed with $n = 2, 3$, and 4 with similar arguments by iterating from lines 6 to 4. Finally, we obtain

$$\begin{aligned} b_{rk_pr}(4) &< 1 \\ s_{j_rr}(4) &> 0 \\ b_{k_tt}(4) &< 1 \\ s_{j_rt}(4) &> 0 \\ s_{j_rm}(4) &> 0 \end{aligned}$$

and

$$\begin{aligned} b_i(4) &< b_i(3), \qquad i = k_r r, rk_p p, k_r m, j_r, k_t m, j_p \\ s_i(4) &> s_i(3), \qquad i = rk_p, j_r r, j_p p, k_t, j_p m, k_r. \end{aligned}$$

The base case is proved. Assume now that $n > 0$

$$s_i(n) > s_k(n-1), \quad i \in I_s, \quad b_i(n) < b_i(n-1), \quad i \in I_b. \quad (26)$$

Then, from Lemma 2 ,we obtain

$$s_i(n+1) > s_k(n), \quad i \in I_s, \quad b_i(n+1) < b_i(n), \quad i \in I_b.$$

Therefore, $s_i(n)$ and $b_i(n)$ are monotonically increasing or decreasing, respectively. Since they are bounded by 0 and 1 [14], they are convergent.

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Productivity of Parallel Production Lines With Unreliable Machines and Material Handling

Theodor Freiheit, Yoram Koren, and S. Jack Hu

***Abstract*—Using parallelism in bufferless production lines can improve productivity, with significant productivity gains achieved with crossover. However, including crossover in the line implies additional material-handling requirements that may reduce the availability of the system. This paper examines if parallel systems with crossover between the stages are more productive than parallel systems without crossover between the stages, when one considers the availability of the additional material handling required for the crossover. The minimum material-handling availability necessary for inclusion of crossover is determined for a given parallel line's configuration such that productivity can be maximized.**

***Note to Practitioners*—Two approaches in configuring parallel manufacturing lines are currently being used in industrial plants. These have been characterized as the Japanese approach of parallel independent cells of serial operations, and the European approach of a serial line with each operation being duplicated in parallel. The European approach has a productivity advantage over the Japanese approach when considering machine failures within each operation. However, the European approach requires more material handling which increases the configuration complexity and can reduce productivity. A math model is developed to determine which approach is best for a given line design when line length is defined by process planning and line balancing, and line width is determined by throughput requirements. The analysis is limited to cell configurations that do not use buffers internal to the cell.**

***Index Terms*—Availability, material handling, productivity, system analysis and design.**

I. INTRODUCTION

Configuration is an important, sometimes overlooked, aspect of the manufacturing-system design that can significantly effect its performance. Its effect has been studied by Koren *et al.* [1] who noted its impact on such parameters as reliability, productivity, quality, scalability, convertibility, and cost. For manufacturing-system design decisions involving capital expenditures, one of the most important parameters is productivity. Traditionally, system productivity is estimated from the availability of the system elements. In automated machining transfer lines, and to a lesser extent in assembly lines, productivity shortfalls due to equipment failures are customarily addressed by the inclusion

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of buffers. However, the trend toward lean manufacturing, with its pull production control philosophy and just-in-time inventory levels, implies a need and desire to reduce and eliminate buffers, especially in industries such as computer chip manufacturing where inventory is very expensive. Parallelism in production stages is another method to improve system productivity without the inclusion of buffers [2].

System parallelism can be realized by designing independent serial production lines and duplicating them in parallel, each with an independent material-handling system servicing all its stations. Parallelism can also be realized in the operations by designing the system as a series of stages of duplicate stations placed in parallel. Here, a common material-handling system transfers the work-in-process between stages, and each stage has additional material handling to transfer the work-in-process to each station, i.e., permits crossover between the serially equivalent lines. Both these configurations can be classified as parallel-serial configurations [3], and can include hybrid combinations of both serial and parallel elements. Because of the potential for station failures, significant improvements to productivity are obtained from the crossover between the production stages by taking advantage of the duplication of operations.

In order to facilitate this crossover, flexible material handling is required to distribute work-in-process between the parallel stations. However, the increased flexibility and control required for this distribution implies greater complexity and associated potential for breakdowns in material handling, and a subsequent impact on system productivity. Uncertainty concerning this tradeoff between the additional complexity of material handling and the magnitude of improvement to productivity when crossover is present has resulted in different manufacturing corporations implementing one or the other of the two different parallelism strategies. Therefore, there is a need to understand which approach is more productive when considering both the increase in productivity due to crossover, and the decrease due to additional material-handling complexity.

Material handling and its influence on the productivity or effectiveness of a manufacturing system has been extensively studied. Much of this research is on the performance of automated guided vehicles in flexible manufacturing systems. Johnson and Brandeau [4] conducted a survey that examined published research on the design and control of automated storage and retrieval, and guided vehicle systems. Most research has concentrated on scheduling and resource allocation such as the number of automated guided vehicles (AGVs) necessary to minimize service wait time, selection of service nodes, or partitioning of networks. Examples include examining scheduling policies on the performance of serially configured, duplicate stations [5], determining the number of work centers to be serviced by an automated component delivery vehicle [6], and determination of an optimal AGV fleet size for an flexible manufacturing system (FMS) when keeping the number of empty trips minimal [7]. Very few published papers model the material-handling system as having a potential for failure, although Beamon [8] cites reliability as an important performance measure. An exception is Beschorner and Glüer [9], who model network bottlenecks when material system elements fail. However, their model does not include the probability that a particular system element fails. Another exception is Savsar [10], who examined the performance of a single flexible manufacturing cell with unreliable production machine and material handling. There is a need to understand the role of unreliable material handling in system performance in complicated system configurations.

This paper models bufferless parallel–serial systems with and without crossover using combinatorial algebra. The unreliability of the system elements, as repairable systems, is modeled through their availability. As a result, the models aggregate all failures into time-based failures, and operationally-based failures are not distinguished. Moreover, the model addresses paced configurations with single process plans, typical of automotive machining lines and other similar manufacturing. Following the introduction, the modeling approach is presented. Next, several cases are presented and discussed. A summary of the design recommendations concludes the paper.

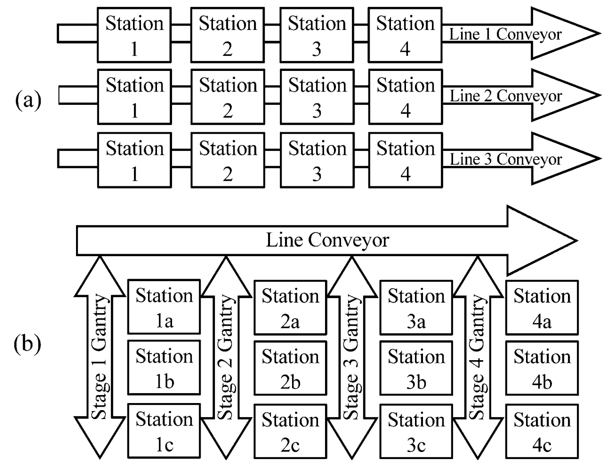

Fig. 1. Schematic example of physical layout of (a) parallel lines and (b) parallel stations.

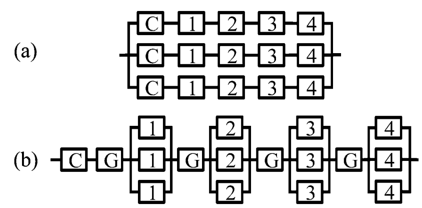

Fig. 2. Schematic logical layouts of (a) parallel lines and (b) parallel stations.

II. Approach

An example of the physical layout of twelve machines configured into three parallel lines or three parallel stations is given in Fig. 1. In the parallel lines configuration [Fig. 1(a)] each line of four stations is serviced by its own conveyor, while in the parallel stations configuration [Fig. 1(b)] each set of three stations is serviced by its own gantry, while material handling between the stages is conducted through a line conveyor. A logic diagram of system elements for the two examples is given in Fig. 2. Each conveyor or gantry is treated as a serial element, as their failures can be modeled as a common cause failure to all stations in an independent parallel line or to the parallel stations system.

The productivity of different system configurations is predicted by determining its productive system states. Since the system is paced, it produces at the rate of the slowest station or stage in a given state. The productivity of a configuration is defined as the normalized expectation of the production rate of all system states. Productivity P, also called system availability or effectiveness, can be mathematically expressed as [3]

$$P = \frac{1}{\mu_{\max}} \sum_{i=1}^{s} \mu_i \Pr(i\text{th state}) \tag{1}$$

where s is the number of system states, μ_i is either zero for a nonproductive state or the production rate associated with the ith state, $\mu_{\max}$ is the highest production rate of the states, and Pr(ith state) is the probability that the ith state occurs.

Applying (1), the productivity of the parallel lines configuration is determined by summing all permutations of the probability that one or more lines are operational and scaling it for its effective production rate. The production rate is scaled to normalize the production rate to one when it is at its maximum, i.e., when all lines are functional. Scaling is achieved in parallel systems by multiplying by the ratio of the equivalent number of functioning lines to total number of lines, e.g., i/m in (2). When all stations have the same availability A then, $(1-A^n)$ is the probability of any one line being down. With m lines in parallel, each with n serial stations or stages, the productivity is

$$P_{\parallel \text{ lines}} = \sum_{i=1}^{m} \binom{m}{i} (1-A^n)^{m-i} A^{\text{ni}} \frac{i}{m} = A^n. \tag{2}$$

Note that placing serial lines in parallel does not in, and of itself, improve productivity, although it does reduce productivity variance [2]. As can be seen in Fig. 2(a), the influence of material-handling availability on the parallel lines configuration is an additional serial component, giving the productivity with material handling as

$$P_{\parallel \text{ lines}} = A_c A^n. \tag{3}$$

Generalizing (3) for each stage (operation) having a unique availability, the productivity is modeled

$$P_{\parallel \text{ lines}} = A_c \prod_{i=1}^{n} A_i. \tag{4}$$

The productivity of a parallel station's configuration is determined by summing all the permutations of the probability of having functioning stations in each stage and scaling it by the effective production rate of the bottleneck stage. When all stations have the same availability and production rate, the productivity is given by

$$P_{\parallel \text{ sta}} = \frac{1}{m}(1-A)^{nm} \sum_{k=1}^{m} \sum_{a_1=k}^{m} \cdots \sum_{a_n=k}^{m} \cdot \binom{m}{a_1} \cdots \binom{m}{a_n} \left(\frac{A}{1-A}\right)^q, \quad \text{with } q = \sum_{j=1}^{n} a_j \tag{5}$$

where the number of stages is n and the number of stations in each stage is m. The summation with index k accounts for the bottleneck that occurs from the minimum number of functioning stations in a stage across all stages.

As can be seen in the logic diagram of Fig. 2(b), the material handling for the parallel stations configuration is modeled by including a single station between each stage that represents the gantry, or other material handler such as a robot or conveyor (here, forward is referred to generally as a gantry). Since each stage is serviced by its own gantry, and the gantries and line conveyors function as serial elements, the productivity of the parallel stations configuration with material handling is modeled

$$P_{\parallel \text{ sta}} = A_c (A_g)^n \frac{1}{m}(1-A)^{nm} \sum_{k=1}^{m} \sum_{a_1=k}^{m} \cdots \sum_{a_n=k}^{m} \cdot \binom{m}{a_1} \cdots \binom{m}{a_n} \left(\frac{A}{1-A}\right)^q, \quad \text{with } q = \sum_{j=1}^{n} a_j \tag{6}$$

where A_g is the gantry and A_c is the line conveyor availability. Equation (6) can be generalized for unique stage availability, giving a productivity model

$$P_{\parallel \text{ sta}} = A_c (A_g)^n \sum_{a_1=1}^{m} \cdots \sum_{a_n=1}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} \times \left(\prod_{j=1}^{n} (1-A_j)^{m-a_j}\right) \left(\prod_{j=1}^{n} A_j^{a_j}\right) \frac{\min(a_1, \ldots, a_k)}{m}. \tag{7}$$

Examining (3) and (6), both parallel line's and parallel station's configurations have a conveyor availability. If the conveyor availability is the same for an independent line as for stage-to-stage material transfer in the parallel stations, (3) and (6) can be set equal and the gantry availability necessary to provide equivalent productivity for the two types of lines can be calculated as shown in (8) at the bottom of the page.

Equation (8) can be generalized for unique stage availability by using (4) and (7), giving the minimum gantry availability for equivalent productivity as

$$A_g^* = \left[D^{-1} \prod_{i=1}^{n} A_i\right]^{1/n}$$

where

$$D = \sum_{a_1=1}^{m} \cdots \sum_{a_n=1}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} \left(\prod_{j=1}^{n} (1-A_j)^{m-a_j}\right) \times \left(\prod_{j=1}^{n} A_j^{a_j}\right) \frac{\min(a_1, \ldots, a_k)}{m}. \tag{9}$$

An approximation of the contribution of variation in the availability of the stations, gantries, and conveyors can be factored into the productivity equations by applying a Taylor Series expansion, known as the propagation of error technique [11]

$$\phi = \phi(x_1, x_2, \ldots, x_j)$$
$$\mu_\phi \cong \phi\left(\mu_{x_1}, \mu_{x_2}, \ldots, \mu_{x_j}\right) + \frac{1}{2} \sum_{i=1}^{j} \frac{\partial^2 \phi}{\partial x_i^2} \sigma_{x_i}^2 \tag{10}$$

where ϕ is a function of a set of independent variables each with a mean μ and a standard deviation σ. This approximation is sufficient for a small variation, a coefficient of variation of $C \lesssim 0.2$. Treated individually, variation in each station, gantry, or conveyor will have no effect on the overall productivity, as the second partial derivative of (7) or (9) is zero. However, if the availabilities of the stations or gantries are co-dependent, it can have a major impact on productivity. For example, if they share a common repair resource such as repair personnel whose presence varies from day to day, e.g., day versus evening staffing levels or absenteeism, all stations have a common variation in availability.

Applying (10) to (3), the mean productivity of parallel lines with station and conveyor variation is

$$P'_{\parallel - \text{lines}} = A_c A^n \left[1 + \frac{n(n-1)}{2} C_A^2\right] \tag{11}$$

$$A_g^* = \left[\frac{A^n}{\frac{1}{m}(1-A)^{nm} \sum_{k=1}^{m} \sum_{a_1=k}^{m} \cdots \sum_{a_n=k}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} \left(\frac{A}{1-A}\right)^q}\right]^{1/n}, \quad \text{with } q = \sum_{j=1}^{n} a_j \tag{8}$$

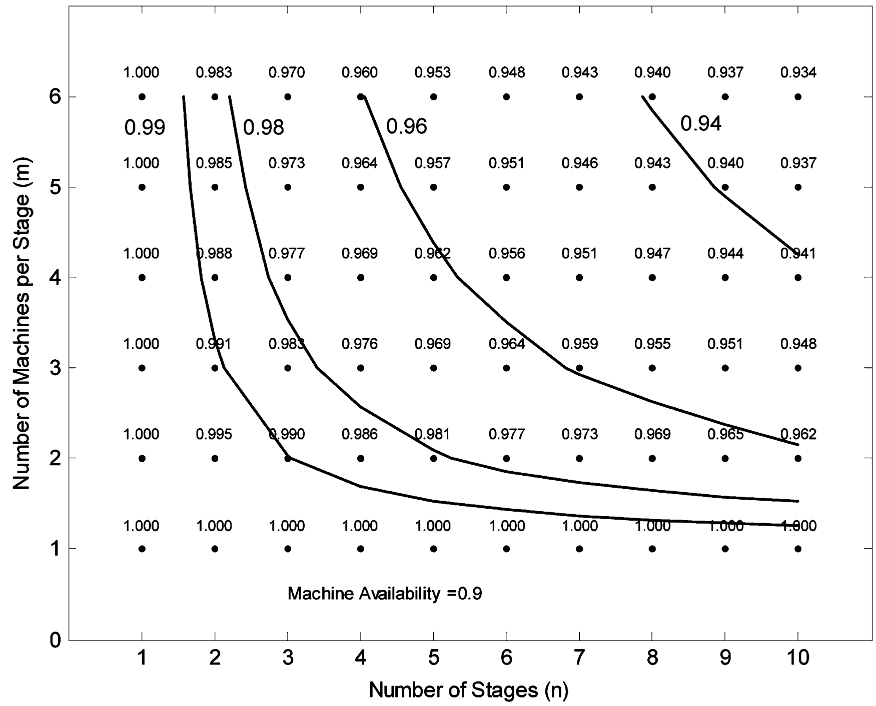

Fig. 3. Minimum gantry availability required for equivalent productivity in configurations with and without crossover.

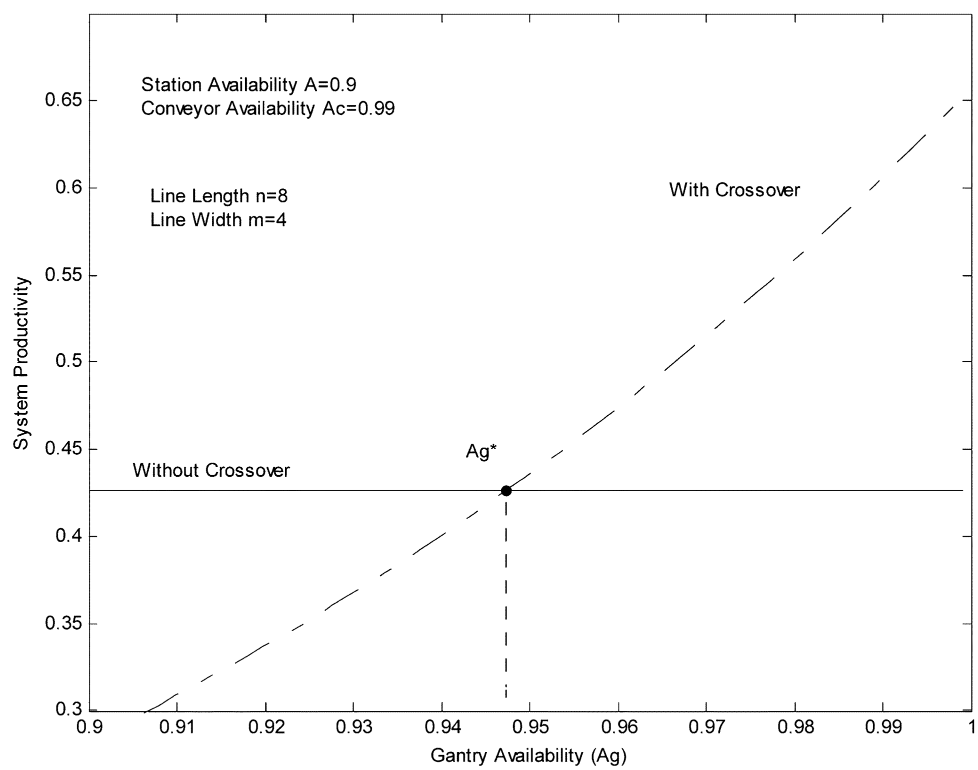

Fig. 4. System productivity at different gantry availabilities.

where C_A is the coefficient of variation of the stations. Note that (11) implies that variation in the mean increases the productivity of a system. This is because the exponent in (3) magnifies the effect of changes in availability, giving proportionally more productivity to increases in availability than losses in productivity with decreases in availability. In practice, this is offset somewhat by the lower mean availability implied by combining both day and evening shifts than when compared to using only the day shift availability mean.

Applying (10) to (6), the mean productivity with parallel stations with station, gantry, and conveyor variation is

$$P'_{\| \,\mathrm{sta}} = A_c (A_g)^n \frac{1}{m} (1-A)^{nm} \times \sum_{k=1}^{m} \sum_{a_1=k}^{m} \cdots \sum_{a_n=k}^{m} \left[\binom{m}{a_1} \cdots \binom{m}{a_n} \left(\frac{A}{1-A} \right)^q \right.$$

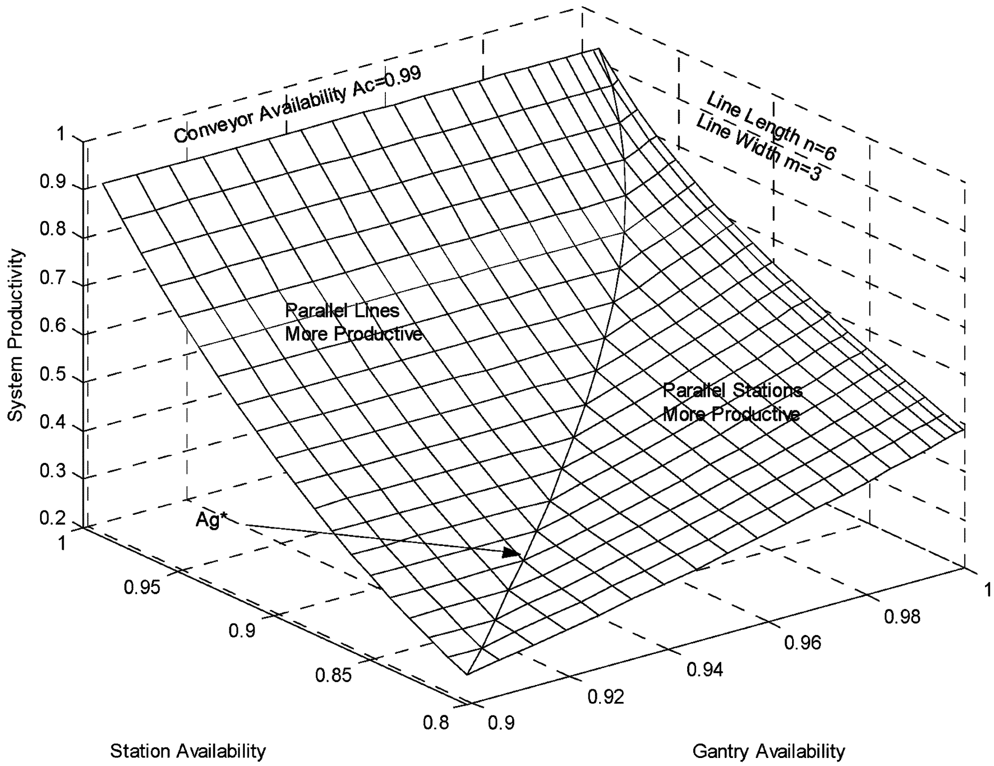

Fig. 5. Productivity tradeoff between station and gantry availability.

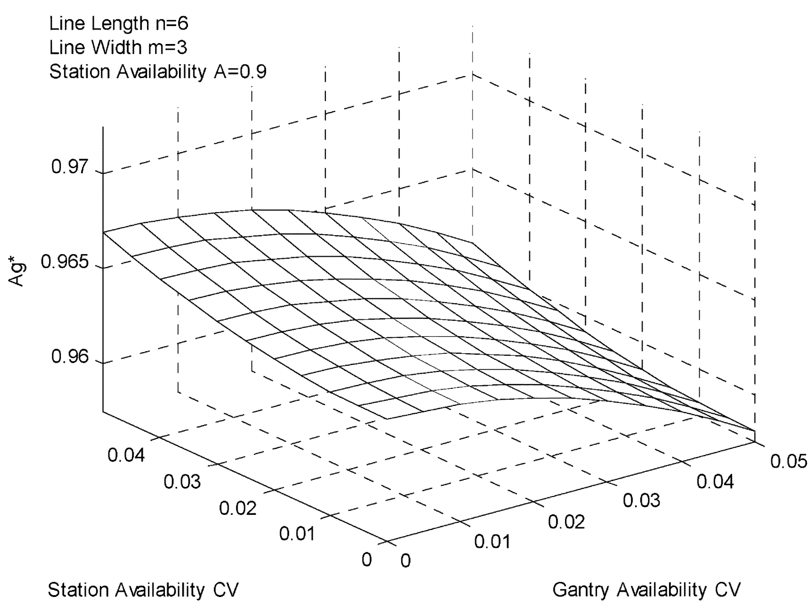

Fig. 6. Minimum gantry availability with station and gantry variability.

$$\times \left(1 + \frac{1}{2}Y_1 C_A^2 + \frac{n(n-1)}{2} C_{A_g}^2\right)\Bigg]$$

with

$$q = \sum_{j=1}^{n} a_j$$

$$Y_1 = (1-A)^{-2}[A^2 mn(mn-1) - 2Aq(mn-1) + q(q-1)] \quad (12)$$

where C_{Ag} is the coefficient of variation for the gantry. Note that in both (11) and (12), conveyor availability variation does not influence the mean productivity. Equations (11) and (12) can be used to calculate the required minimum gantry availability, A_g^*, in the same form as (8).

In general, if the material-handling availability exceeds A_g^*, then, a parallel stations strategy will provide higher productivity. Likewise, if the material-handling availability is less than A_g^*, then, a parallel lines strategy (no crossover) will provide higher productivity.

III. Results and Discussion

Whether to include crossover in a configuration to maximize its productivity can be determined by comparing the availability of the crossover material-handling system to the A_g^* of the system configuration. Fig. 3 plots contour lines for the minimum gantry availability required for different line configurations to benefit from crossover in improving productivity. For illustrative purposes, the figure plots (8) with a constant station availability of 0.90. The x-axis is the number of stages, while the y-axis is the number of lines or stations in parallel. To use this plot, determine the system's gantry availability and locate the line associated with it on the plot. If the configuration length-width point falls below left of the contour line labeled with the gantry availability, then a parallel lines (no crossover) configuration provides greater productivity. If the configuration length-width point falls above right to the contour line, a parallel stations configuration (crossover) will provide greater productivity. For reference, a typical gantry availability taken from actual industrial experience is 0.98, representing a minor repair of 15 min once a day, and a major repair taking one day occurring once a year. Each configuration length-width point is labeled with its A_g^*, that is, the gantry availability that provides equal productivity between crossover and no crossover configurations.

The influence of gantry availability on the productivity of the system can be seen in Fig. 4, which plots the productivity of a production line of length eight and width four. Each station has an availability of 0.9 and the line conveyor has an availability of 0.99. When the gantry availability is less than A_g^*, the productivity is higher for a no-crossover configuration (which do not require gantries). When gantry availability is greater than A_g^*, a crossover configuration has better productivity. As can be seen, at high levels of gantry availability, parallel configurations that do not include buffers have significant increases to productivity that are gained from the presence of crossover.

Fig. 5 shows the relationship between station availability and gantry availability for a line of length $n = 6$ and width $m = 3$. As is expected, at low gantry availability, parallel-line configurations are more productive. Further, at high station reliability, very high gantry availability is necessary for the parallel-lines configuration to provide higher productivity. Fig. 5 implies that in parallel station configurations, if the stations in a given stage have a lower availability compared to their neighbors, a good design strategy is to insure that such stages are serviced by a gantry with high availability to increase overall system productivity.

Fig. 6 plots A_g^* for a line of length $n = 6$, width $m = 3$, and a mean station availability of $A = 0.9$, when the coefficient of variation of the station and gantry availability is varied. This plot implies that if the gantry availability varies greatly from day-to-day, lower gantry availability is necessary to achieve higher productivity with crossover, and therefore a parallel stations strategy is more effective. Likewise, if the station availability varies greatly from day-to-day, the higher gantry availability required implies a parallel lines strategy is more effective. The best strategy when both station and gantry availability variation is high is a parallel stations configuration, as the gantry variation is more dominant.

IV. Conclusion

To take advantage of parallelism in a bufferless manufacturing system, a chief design objective should be to maximize material-handling equipment availability. Without highly available gantries, the significant productivity gains that are achieved from crossover cannot be obtained. Design considerations should include accessibility to down stations for safe repair without interrupting the gantry and its service of other stations within the stage, appropriate selection of material-handling technologies to achieve the desired availability relative to the station availability, and the relinquishing of crossover as unnecessary when sufficiently high station availability can be designed into the system.

Note that this analysis does not address the economic tradeoff to improved productivity. The additional cost of gantries, while providing a greater productivity, may not be justifiable for the additional profit that they make possible. Finally, an additional benefit of parallel stations that has not been part of the models in this paper is the inherent ability of parallel lines to provide the right scale of production under a changing demand environment. Adding stations in parallel at the bottleneck operation can address shorter term throughput requirements more feasibly than the addition of a parallel line. System designers should consider this possibility in determining its gantry specifications, allowing for modular or expandable material handling, especially if sales growth is anticipated over the life cycle of the product.

Acknowledgment

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