INT. J. PROD. RES., 15 MAY 2004, VOL. 42, NO. 10, 2009–2027



# Productivity of synchronized serial production lines with flexible reserve capacity

T. FREIHEIT†*, M. SHPITALNI§, S. J. HU†‡ and Y. KOREN†‡

The configuration of a manufacturing system greatly influences its productivity because many configurations have multiple productive states. Flexible reserve production capacity is one method to increase the number of productive states. Reserve production capacity is the provision of non-dedicated standby machines in parallel to the main production line that are capable of performing any operation in the production line. Standby machines, like buffers, isolate failures in the production line, permitting production to continue. This paper develops models to predict the productivity of pure serial and parallel-serial production lines with reserve capacity. Combinatorial mathematics is applied to determine the magnitude of production and the probability of occurrence of system states. Productivity improvements are quantified and the productivity equivalency of reserve capacity to buffers is demonstrated.

## 1. Introduction

Productivity (or production effectiveness or steady-state system availability) is an important manufacturing system performance measure that has been traditionally evaluated by examining the manufacturing system component reliability and processing speed. However, manufacturing system configuration, which traditionally has received less attention than these two factors, has a large impact on system productivity because many configurations have multiple productive states (Koren *et al.* 1998). One important aspect of system configuration that has been extensively studied is the placement and size of buffers between manufacturing operations (e.g. Gershwin and Berman 1981). Through buffers, lengthy serial lines continue to produce when some of the system's constituent machines fail, mitigating system failure by temporally decoupling these machine failures from system failure. However, since buffers are relatively inexpensive, other configuration approaches to decouple machine and system failure have not been explored. One such approach is to provide reserve production capacity to a serial-type, synchronized production line without buffers.

Reserve production capacity is defined as the provision of flexible standby machines capable of performing any operation being performed in the main production line. The main production line is a synchronized serial transfer line with or without groups of parallel stations. This type of transfer line, for example, is found in short segments of machining lines that produce automotive components,

Revision received September 2003.

†Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI, USA.
‡Engineering Research Center for Reconfigurable Manufacturing Systems, 2250 GG Brown, 2350 Hayward, Ann Arbor, MI 48109-2125, USA.
§Faculty of Mechanical Engineering, Technion-IIT, Haifa, Israel.
*To whom correspondence should be addressed. e-mail: tfreihei@engin.umich.edu

*International Journal of Production Research* ISSN 0020–7543 print/ISSN 1366–588X online © 2004 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1081/00207540310001647596

where each station is synchronized to a common cycle time associated with the desired rate of production, but limited to the rate of the slowest station. Synchronization is used to minimize the production rate variability seen in queued systems. However, long synchronized production lines are rarely deployed because the coupling of stations causes an unacceptable probability of system failure resulting from any single station failure. Like buffers, the application of reserve capacity effectively decouples individual station failures from system failure.

In reserve capacity, the standby machines are not part of the production effort except when failures occur in the main line. Consideration of reserve capacity in production system design is facilitated by the increasing use of identical, highly flexible CNC (Computer Numerical Control) machine tools. To illustrate, consider a CNC machine failing in the main production line. While being repaired, an identical standby machine is brought on-line by providing fixtures and the appropriate CNC programs to perform the operations of the failed machine. The flow of production material is routed through an automated material-handling system to the standby machine, and returned to the main production line. When the failed main line machine is repaired, product flow returns to normal, and the standby machine is returned to idle, awaiting the next main line failure.

Figure 1(a) illustrates a schematic of a pure serial line with no buffers, but with reserve capacity. Figure 1(b) illustrates a schematic of a parallel-serial line with reserve capacity. For both lines, the automated material-handling system is capable of moving the work-in-process bidirectionally between the main line and the standby machines, which are labelled ‘s’.

In this paper, productivity is defined as a stochastic measure of steady-state system production effectiveness. Productivity is distinguished from throughput (i.e. parts/min), as it provides a normalized measure of the combined contribution from the production rates of the different productive system states found in a given system configuration (Freiheit *et al.* 2002). Productivity is used to assess the desirability of alternative configurations when examining their cost during the design of production systems. The basis of productivity analysis is the steady-state availability of the system's machines as determined by the application of renewal theory. Availability,

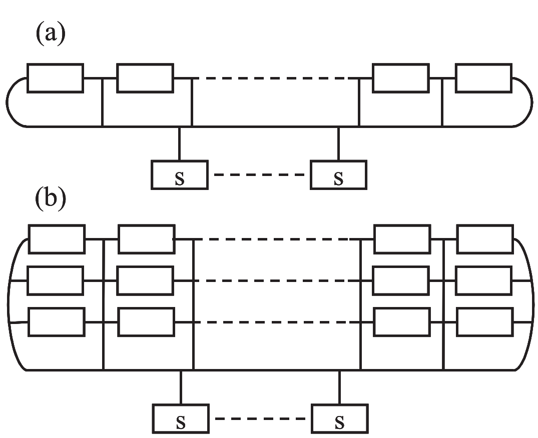


Figure 1. Schematic of reserve capacity lines with a bi-directional material-handling system: (a) pure serial line; (b) parallel serial-line with crossover.

$R$, is the ratio of the mean time between failure (MTBF) over the sum of the MTBF and mean time to repair (MTTR) of the machine:

$$R = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}}. \tag{1}$$

This approach requires the assumption that failure and repair times are individually and independently distributed. The MTTR is also assumed to include any downtime waiting for repair resources. The MTTF encompasses all failure modes and does not limit the availability to operational failures.

The reliability and availability behaviour of standby systems have been extensively studied, as exemplified by the large number of papers cited in a review by Kumar and Agarwal (1980). More recently, in standby systems examining configurations, Kubat and Sumita (1985) developed a model for determining the availability of a serial transfer line machine that has a dedicated standby machine in parallel, then applied it to line design. Both Shen and Xie (1991) and Meng (1995) examined the relative reliability of redundancy placed at a component and system level for serial and parallel systems. Galikowsky *et al.* (1996) examined the reliability and availability of several repairable systems configured with multiple standby units. Wang and Loman (2002) examined the availability of a $k$-out-of-$n$ parallel system with multiple standby units in power generation. However, the influence of standby machines on manufacturing system productivity has not been explored.

Reserve capacity provides an opportunity to reduce or eliminate buffers, yet maintain high productivity. Manufacturing systems that do not rely on buffers are important for reduced inventories and early quality problem detection. Further, reserve capacity systems may be desirable for pull-type operations, where not having buffers is an advantage. This paper examines and demonstrates the synergistic productivity improvements that can be obtained by providing reserve capacity to serial-type production lines. Following the introduction, notation is summarized. In Section 2, models are developed for the estimation of system productivity for pure-serial and parallel-serial lines with reserve capacity. In Section 3, the results and applications of these models are examined and discussed. Conclusions are provided in Section 4. An appendix provides a detailed example of productivity calculation for a pure serial line with operation-dependent machine availability.

### 1.1. *Notation*

- $\alpha$ transfer inefficiency productivity discounting factor,
- $\beta$ multifunctional standby machine productivity discounting factor,
- $\sigma_i$ main line saturation length — point where main line demand peaks,
- $\mu_i$ production rate of the $i$th station,
- $a_i$ number of failed machines in the $i$th operation of a parallel-serial main line, or the index of the $i$th failure,
- $\vec{a}_i$ index set of $i$ failed operations in a operationally dependent serial line,
- $b^{(j)}$ index of the $j$th minimization operation,
- $D_k$ main line demand — probability that up to $k$ failures can occur in a main line,
- $gf$ generating function,
- $k$ number of standby machines,
- $n$ length of main line,
- $m$ number of stations in parallel in each operation of a parallel-serial line,

| | |
|---|---|
| $P$ | system productivity, |
| $q$ | total number of failed machines in a parallel-serial main line, |
| $R'_s$ | probability that a standby machine can cover a failed main line machine, |
| $R_i$ | main line machine or station availability of $i$th operation, |
| $R_s$ | standby machine availability, |
| $u_{\bar{a}}$ | effective number of parallel lines after standby machine replacement of failures in a parallel-serial line. |

## 2. Productivity with reserve capacity

Throughput is typically defined as the long run average production of a system, presented as the number of units produced over a given period (Hopp and Spearman 1996). In the present paper, productivity is the expected throughput of a manufacturing line configuration normalized over the maximum production rate of its system states. The general definition of productivity, $P$, is given by:

$$P = \frac{1}{\mu_{\max}} \sum_{i=1}^{n} \mu_i \text{ prob}(i\text{th state}), \tag{2}$$

where $n$ is the number of system states, $\mu_i$ is either zero for a non-producing state or the production rate associated with the $i$th state, prob($i$th state) is the probability that the $i$th state occurs, and $\mu_{\max}$ is the maximum production rate of all of the states. Note that one can also sum over only the productive states.

The probability that a main line with reserve capacity requires a standby machine to substitute for a failed machine is defined as the demand of the system. The demand, $D_k$, is the probability that up to $k$ standby machines are called upon to compensate for up to $k$ main line failures and that the operation of these standby machines can improve the productivity of the system. It is possible in some system configurations that a standby machine can substitute for a failed main line machine, but its presence provides no improvement to the productivity, as will be shown below. Note that the number of machine failures in the main line at any given time for which the standby machines can substitute is limited by the number of functional standby machines.

There is a possibility that a standby machine will fail and become unavailable for current or future main line failures (until repaired). However, when the number of standby machines exceeds the number of failed machines in the main line, it is possible for a different standby machine to substitute for the main line failure. The probability that there are sufficient standby machines operating to cover main line failure is denoted by $R'_s$, which is a function of the number of standby machines, their availability and the number of failed machines in the main line. Note that a standby machine replaces only one failed machine in the main line during the duration of that main line failure.

The number of productive system states in a serial-type system configuration is a function of the number of standby machines. For a pure serial line, the number of system states equals the number of standby machines plus one, since without standby machines, there is only one productive system state. The probability that productive system states will occur is given by whether a standby machine is in demand (a failure has occurred in the main line) and available (a standby machine is operational). This probability is a function of $D_k$ and $R'_s$.

This analysis assumes that the production system is continuously monitored, there are no failures in the switching decision, nor are there any 'hidden' failure modes. It is also assumed that the production line is synchronized. That is, the production rate is identical and constant at each main line operation (no variability) and all material handling occurs within the cycle time. The standby machine is on hot idle such that at any time it may be in an operational or failed condition (under repair). This assumption is reasonable in calculating productivity because a standby machine that becomes unavailable while substituting for a main line machine may continue to be unavailable after the main line machine for which it had substituted is repaired. It is assumed that there are sufficient repair facilities and that repair brings the machine to a like-new state. All machines in the system have independent failure and repair distributions.

Models for three cases of reserve capacity are developed: a pure serial line with uniform operational availability (all machines in the main line have the same availability), a pure serial line with operation dependent availability (every machine has an availability dependent upon the operation it is performing) and a parallel-serial line with uniform operational availability. The model first develops the demand of the main line and then develops the standby availability. Finally, the productivity is derived from the operational states.

### 2.1. *Productivity of pure serial lines with reserve capacity*

Two models are developed to predict the productivity of a pure serial configuration (e.g. figure 1(a)). The first model assumes that all operations in the main line have the same availability, while the second model assumes that their availability can differ.

#### 2.1.1. *Uniform operational availability in the main line*

The demand, $D_k$, of a pure serial reserve capacity system in a system of $n$ machines with $k$ standby machines is:

$$D_k = \sum_{i=1}^{k} \binom{n}{i} R^{n-i}(1-R)^i, \tag{3}$$

where $R$ is the availability of a machine in the main line. Thus, the demand represents the probability of all possible ways that 1–$k$ out of $n$ machines in the main line have failed. The standby coverage probability, $R_s'$, for $k$ standby machines and $i$ failed main line machines is:

$$R_s' = \sum_{j=i}^{k} \binom{k}{j} R_s^j (1-R_s)^{k-j}, \tag{4}$$

where $R_s$ is the availability of the standby machine. The availability of the standby machine can be considered different from the main line machines due to the multi-functionality of the standby machines. The summation is used to determine the probability that $i$ out of $k$ standby machines are available. Combining equations

(3) and (4), the productive system states of $i$ possible failed main line machines that are covered by the $k$ standby machines has probability:

$$\text{prob}(i \le k) = \sum_{i=1}^{k} \binom{n}{i} R^{n-i}(1-R)^i \left( \sum_{j=i}^{k} \binom{k}{j} R_s^j (1-R_s)^{k-j} \right). \quad (5)$$

The productivity of the reserve capacity system is derived from equation (5) by summing the productivity when all $n$ main line machines are functioning with the productivity influence of the $k$ standby machines:

$$P_{\text{reserve}} = \mu_m \sum_{i=0}^{k} \binom{n}{i} R^{n-i}(1-R)^i \left(1 - \frac{i}{n}(1-\alpha)\right) \left( \sum_{j=i}^{k} \binom{k}{j} R_s^j (1-R_s)^{k-j} \right), \quad (6)$$

where $\mu_m$ is the production rate of the main line when all machines are functioning, generally normalized to 1, and $\alpha$ is a productivity-discounting factor associated with the efficiency in the transfer from the main line to the standby machines. This productivity discounting factor, set to 1 for when the transfer to the standby machine is completely efficient, e.g. within the main line cycle time, is scaled linearly with the number of failures in the main line divided by the length of the main line. This scaling is justified by traffic congestion during transfer in shorter lines. As appropriate, $\alpha$ can be considered a scaling factor for an increase in the serial line synchronized cycle time to accommodate the transfer time to the standby machine.

Standby demand is useful for determining the number of standby machines necessary to serve a given main line length adequately. Standby machines become overwhelmed by failures in the main line at a point when the number of failures in the main line is greater than the number of standby machines. This point, called the saturation length, $\sigma$, is reached when the demand for standby machines no longer increases. After this point, the probability that the number of failures in the main line is less than or equal to the number of standby machines begins to decline, and the main line length should not exceed this saturation point. The saturation point can be derived by taking the derivative of the demand equation and setting it equal to zero. However, for the general case, no closed form solution exists and $n$ must be solved graphically, or numerically from equation (7):

$$\begin{aligned}\sigma_n &= \frac{\mathrm{d}}{\mathrm{d}n} D_k \\ &= \frac{R^{n-1}}{(k+1)!(n-k-1)!} \left[ \begin{array}{l} \left\{ R(k+1)!(n-k-1)! \left( \left(\frac{1}{R}\right)^n \ln\left(\frac{1}{R}\right) + \left[ \left(\frac{1}{R}\right)^n - 1 \right] \ln(R) \right) \right. \\ + \left(\frac{1}{R} - 1\right)^k (R-1) n! \, {}_2F_1\left( \begin{matrix} 1, 1+k-n \\ 2+k \end{matrix}, \frac{R-1}{R} \right) \\ \left. \times \left( \ln R + \sum_{i=0}^{\infty} \frac{1}{n+1+i} + \sum_{i=0}^{\infty} \frac{1}{n-k+i} - 1 \right) \right\} \end{array} \right] = 0, \quad (7)\end{aligned}$$

where ${}_2F_1\left( \begin{matrix} a, b \\ c \end{matrix}, z \right) = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}$ is the hypergeometric function and $(a)_n$ is the Pochhammer symbol (rising factorial). In two cases, when the number of standby machines is one or two, a closed form solution exists. The main line saturation length, $\sigma_1$, for one standby machine is:

$$\begin{aligned} D_{k=1} &= n(1-R)R^{n-1} \\ \sigma_1 &= \frac{\mathrm{d}}{\mathrm{d}n} D_{k=1} = (1-R)R^{n-1} + n(1-R)R^{n-1}\ln R = 0 \\ \therefore \sigma_1 &= -\frac{1}{\ln R}. \end{aligned} \tag{8}$$

The main line saturation point for two standby machines is:

$$\sigma_2 = \frac{2 - 2R + (3R-1)\ln(R) + \sqrt{4(R-1)^2 + (1-3R)^2(\ln R)^2}}{2(R-1)\ln(R)}. \tag{9}$$

### 2.1.2. *Operation-dependent availability in machines*

The productivity of a serial production system where the availability of all machines, or operations, are dependent on the operation being performed is complicated by how failures are distributed along the main line. The demand for a pure serial line with operationally dependent machine availabilities is:

$$D_k = \sum_{i=1}^{k} \sum_{a_1=1}^{n-i+1} \sum_{a_2=a_1+1}^{n-i+2} \cdots \sum_{a_i=a_{i-1}+1}^{n} \left[ (1-R_{a_1})\cdots(1-R_{a_i}) \prod_{j \notin \vec{a}} R_j \right], \tag{10}$$

where $\vec{a}_i$ is a vector of length $i = 1, \ldots, k$ representing the index set of failed operations, $a_0 = 0$, and $R_{a_i}$ is the availability of the $a_i$th operation. Equation (10) is derived by summing all possible distributions of up to $k$ failures in a serial production line. A specific example is given in the appendix of the application of equations (10–14).

The probability that at least $k$ standby machines are available can be determined by finding all combinations of failed and operational standby machines that provide at least the same number of functional back-up machines as failed machines in the main line. The mathematics of this is complicated by situations where the number of standby machines exceeds the number of failed main line machines, and the standby machine could replace both a failed main line machine and a failed standby machine. One method of determining this probability, which separates out the 'standby for a standby' machine, is to apply generating functions. Generating functions are a formal algebraic object developed by Euler for combinatorial analysis. Here, it aids in establishing the combinations of standby machine failures that are permitted to occur when there are more standby machines than main line machine failures by enumerating all of the ways in which this can happen. Let $b_q$ be the coefficient of the $q$th exponential term of the generating function $gf$ when expanded:

$$gf = \prod_{p=1}^{i} \left[ \sum_{q=0}^{k-i} (1-R_{a_p})^q x^q \right], \tag{11}$$

where $i \leq k$ is the number of failed machines, $a_p$ is an index of an operation that has failed in the main line, $k$ is the number of standby machines and $x$ is a 'place holder' for the generating function. This generating function can also be thought of as the convolution of all $i$ vectors whose elements are the failure probabilities to the $q$th power. The probability that sufficient standby machines are functioning is then:

$$R'_{si} = \left\{ \prod_{p=1}^{i} R_{a_p} \right\} \sum_{q=0}^{k-i} b_q \qquad (12)$$
$$R'_{si} = 1 \quad \text{when } i = 0,$$

where $R_{a_p}$ is the availability of the operations on the standby machines that are replacing the failed operations in the main line, and the summation represents the back-up of the excess standby machines to the standby machines. Combining equations (10) and (12), the productivity of an operation-dependent serial production system with reserve capacity is:

$$P_{\text{reserve}} = \mu_m \left\{ \prod_{i=1}^{n} R_{a_i} + \sum_{i=1}^{k} \left( 1 - \frac{i}{n}(1-\alpha) \right) \times \sum_{a_1=1}^{n-i+1} \sum_{a_2=a_1+1}^{n-i+2} \cdots \sum_{a_i=a_{i-1}+1}^{n} \left[ (1-R_{a_1}) \cdots (1-R_{a_i}) \left( \prod_{j \notin \vec{a}} R_j \right) R'_{si} \right] \right\}, \qquad (13)$$

where $a_0 = 0$ and once again $\alpha$ is a productivity discounting factor for transfer efficiency.

Note that in both equations (6) and (13), the production rate, $\mu_m$, is the bottleneck production rate of all of the operations. That is, the production rate of the slowest operation inclusive of material-handling time, normalized to 1. Equation (13) can also be modified to provide a reduction in the availability of the multi-capable standby machines, revising the generating function and standby machine probability to:

$$gf = \prod_{p=1}^{i} \left[ \sum_{q=0}^{k-i} (1 - \beta_{a_p} R_{a_p})^q x^q \right]$$
$$R'_{si} = \left\{ \prod_{p=1}^{i} \beta_{a_p} R_{a_p} \right\} \sum_{q=0}^{k-i} b_q, \qquad (14)$$

where factor $\beta_{a_p}$ is an additional serial availability associated with the $a_p$th operation.

### 2.2. *Productivity of parallel-serial lines with reserve capacity*

A parallel-serial line is defined as a set of $m$ serial machining lines each of $n$ machines, configured in parallel to each other (Freiheit *et al.* 2002). These lines can either have crossover between every operation (i.e. have serial groups of operations) or have no crossover and be completely independent of each other. Crossover is where a product manufactured on a line blocked due to a failure upstream can be transferred to another line and products from other lines can be transferred to the blocked line downstream of the failure. With reserve capacity, a parallel-serial line must be designed with crossover; otherwise, transfer to and from the failed machine cannot occur.

2.2.1. *Uniform operational availability in the main line*

The productivity of a parallel-serial line is dependent on the distribution of the functional and failed machines within each operation, where bottlenecks within the operations are caused by the machine failures. Reserve capacity provides a replacement for failed machines, reducing or eliminating the bottleneck(s). With $m$ main lines each of $n$ machines, the number of standby machines is limited to $k < n$. Whereas a limit on the number of standby machines is not a strict requirement, $n$ or more standby machines would be a waste of resources, as a full complement of $n$ standby machines could be used in continuous production as an additional parallel-serial machining line, which itself provides synergistic improvements to productivity (Freiheit *et al.* 2002).

The productivity of a parallel-serial line with reserve capacity differs from that of a pure serial line due to the redundancy of the parallel lines. Unlike a pure serial line, when the number of failures in the main line exceeds the number of back-up machines, additional productive states are present in the parallel-serial line. For a parallel-serial system whose machine availabilities are homogeneous, the demand for a parallel-serial system with reserve capacity of $k$ standby machines is:

$$D_k = \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} (1-R)^{nm-q} R^q, \quad \forall u_{\vec{a}} > \min(a_1, \ldots, a_n), \tag{15}$$

where $q = \sum_{j=1}^{n} a_j$ is the total number of functional machines in each state, and $u_{\vec{a}}$ is determined as follows: let $b^{(j)}$ be the index of the $j$th minimization operation and $b^{(0)} = \min(a_1, \ldots, a_n)$. Then $u_{\vec{a}}$ is:

$$u_{\vec{a}} = \min_{j=0}^{k}(a_1, \ldots, a_{b^{(j)}} + 1, \ldots, a_n), \tag{16}$$

where $a_i$ is the number of functional machines in the $i$th operation, and when $a_i < m$, a bottleneck is present in the main production line. The parameter $u_{\vec{a}}$ is of much interest for a parallel-serial system as it captures system states where the number of failures in the main line exceeds the number of available standby machines, yet functioning standby machines still contribute significantly in improving the productivity of that state by moderating bottlenecks. Further, it reflects that standby machines are allocated to main line failures where they are most effective: addressing the most egregious bottlenecks as a first priority.

For the parallel-serial system, the probability that $i$ standby machines are available is adapted from equation (4):

$$R_s' = \sum_{i=1}^{k} \binom{k}{i} R_s^i (1 - R_s)^{k-i}. \tag{17}$$

Using equations (15–17), the probability for a productive system state is:

$$\text{prob(productive state)} = \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} (1-R)^{nm-q} R^q R_s', \forall u_{\vec{a}} > \min(a_1, \ldots, a_n) \tag{18}$$

and productivity is:

$$P_{\text{parallel-serial reserve}} = \sum_{i=0}^{k} \sum_{a_1=0}^{m} \cdots \sum_{a_n=0}^{m} \binom{m}{a_1} \cdots \binom{m}{a_n} (1-R)^{nm-q} R^q \left( \binom{k}{i} R_s^i (1-R_s)^{k-i} \right) \frac{\beta u_{\vec{a}}}{m}$$

with $q = \sum_{j=1}^{n} a_j$

$$u_{\vec{a}} = \min_{p=0}^{i}(a_1, \ldots, a_{b(p)} + 1, \ldots, a_n).$$

Note that the productivity equation includes the contribution for both states where no standby machines are functioning, and for states where failure exist in the main line yet the standby machines do not improve productivity. The normalization assumes that every serial line in parallel contributes $1/m$ to the production rate, giving normalized $\mu_m = 1$ when all machines are functioning, and a standby machine production rate discount factor of $\beta$.

## 3. Results, discussion and applications

Systems with reserve capacity generally require greater capability than a traditional production system. The system must have a material-handling system that can move the part from the main line to a standby machine, and back again, and the standby machine must be capable of performing all of the operations in the main line. Thus, the standby machine can be considered to have a different availability than the main line machines, as the availability encompasses the material-handling system, and the possibility that the standby machine is more complex. Further, the production rate of the standby machines can be considered different due to the additional material transfer time necessary to move the product from the main line to the standby machines, and any production inefficiencies associated with the multicapability of the standby machine.

Figure 2 illustrates the productivity of a serial line with one to three standby machines. As would be expected, as the number of machines in the main line is increased, the productivity of the system falls. However, the redundant duty of the standby machines permits a lesser rate of loss of productivity as the main line is lengthened. This is especially true with more standby machines. Further, even when the standby machines are modelled with less availability, significant productivity improvements are still achieved. Finally, figure 2 shows there is a diminishing return with the addition of each standby machine — the largest improvement is seen with the first standby machine. Figure 3 examines the effect of machine availability on productivity of a pure serial line of six machines. Plotted is the ratio of the system productivity with standby machines to the productivity of the system without standby machines. Figure 3 indicates that the value of standby machines is much greater for systems where the individual machines are less available. However, significant productivity gains, greater than 20%, are obtained even at individual machine availabilities of 95%, in this six-machine main line. This plot also shows that as machines become more available, the additional contribution of each standby machine is diminishing.

The productivity performance of parallel-serial machining lines is similar to the pure serial line, again with productivity improvements. Figure 4 shows the

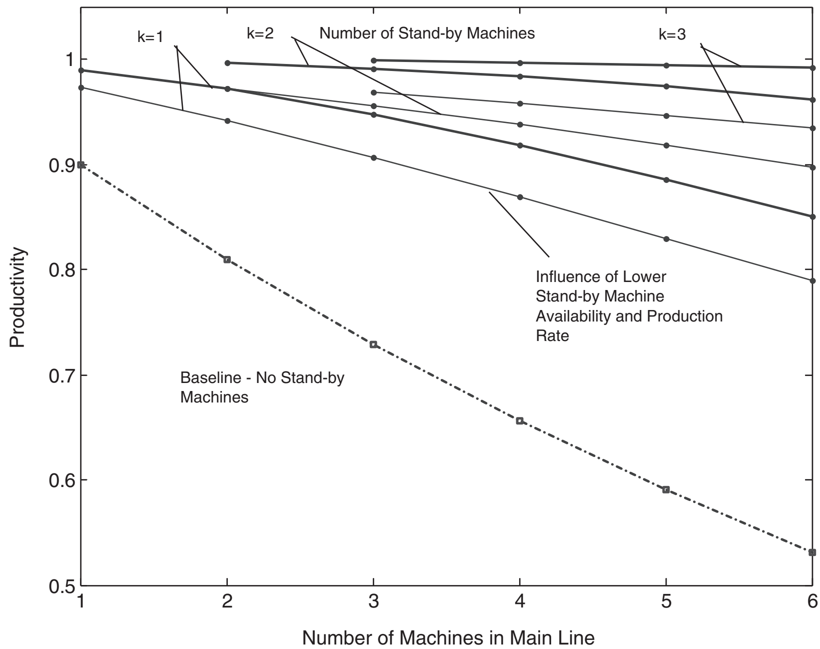


Figure 2. Productivity of a pure serial line with a reserve capacity machine availability $R = R_S = 0.9$, production rate $\mu_m = \mu_s = 1$ and reduced standby machine availability $R_S = 0.81$, $\mu_b = 0.9$.

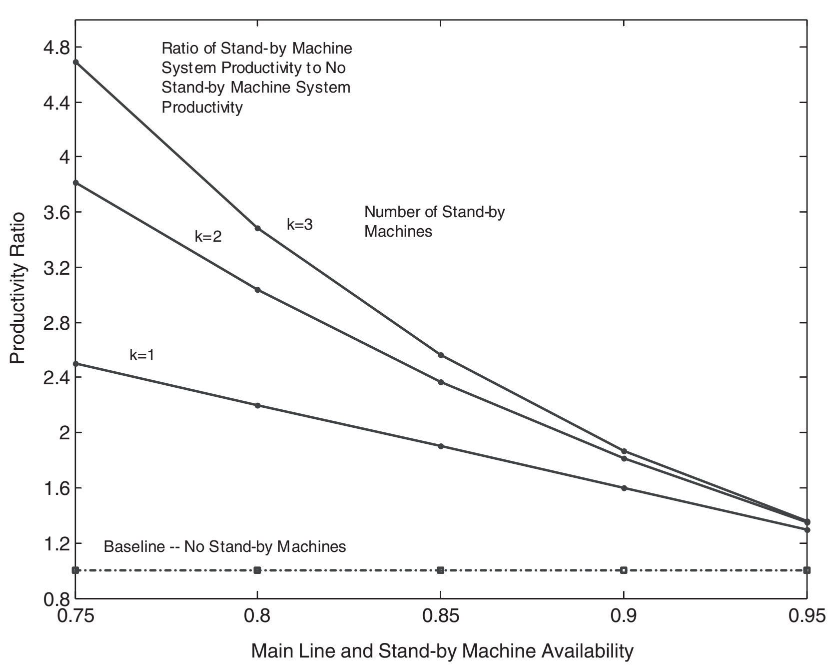


Figure 3. Productivity of a pure serial line as machine availability is varied.

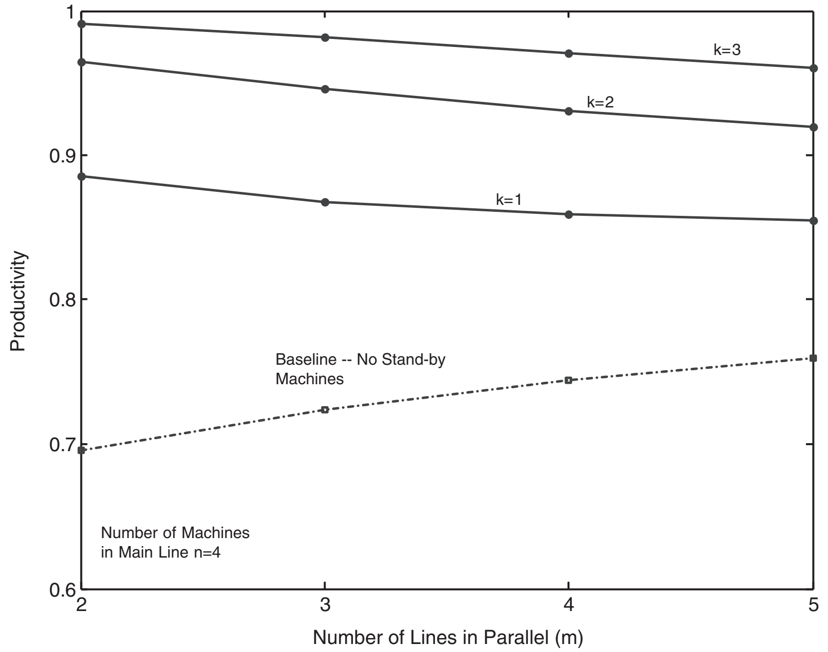


Figure 4. Productivity of a parallel-serial with reserve-capacity four-machine main line with an increasing number of lines in parallel.

productivity trends of parallel-serial lines as the number of parallel lines is increased. In this plot, each main line has four machines. As the number of lines in parallel is increased, the productivity of a system without standby machines increases. This results from the increase in flow paths (productive states) available in parallel systems. However, with reserve capacity, while having greater overall productivity, the addition of parallel lines actually decreases the system productivity. This is due to the rapid increase in the total number of machines each additional parallel line provides, increasing the probability of failure in the main line, and thus overwhelming the standby machines. Again, the diminishing returns to productivity of increasing the number of standby machines can be observed.

Figure 5 shows the productivity trend as the number of machines in each main line is increased when the number of parallel lines is held constant at four. Now, however, the system productivity with reserve capacity is decreasing with increasing line length, as adding length to the line does not add additional production paths. Here too, the standby machines become progressively more overwhelmed with additional line length. Additionally, with standby machines, there is a greater rate of decrease in system productivity with the addition of line length than with the addition of another line in parallel. This is due to the better ability of the standby machines to mitigate production bottlenecks when more flow paths are available.

An examination of the demand for standby machines finds that the number of machines in the main line at saturation is less for pure serial lines (figure 6) than for parallel-serial lines (figure 7). This is again due to the better ability of the standby machines to mitigate production bottlenecks of the multiple flow paths available in the parallel-serial configuration, improving the production rate of more system states

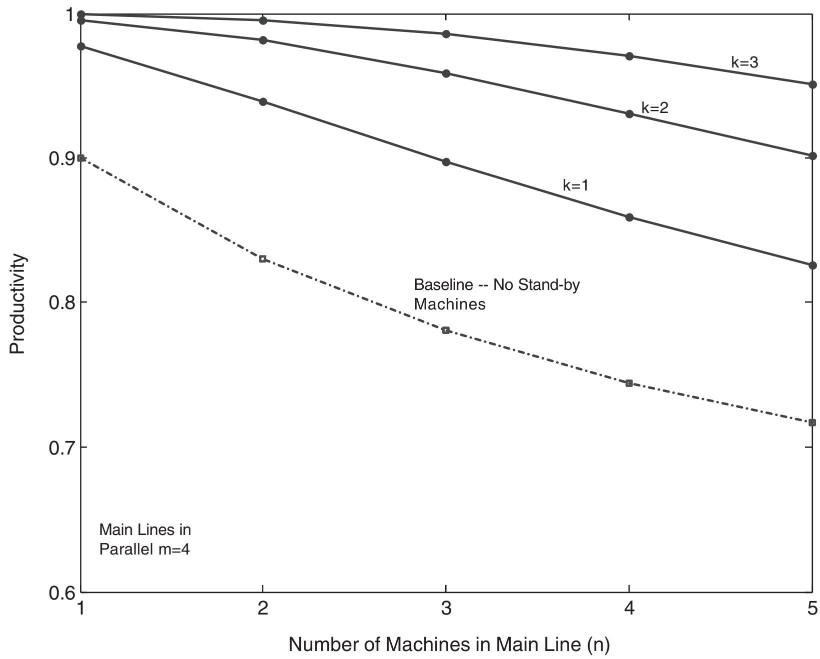


Figure 5. Productivity of parallel-serial with reserve-capacity four main lines in parallel with increasing main line length.

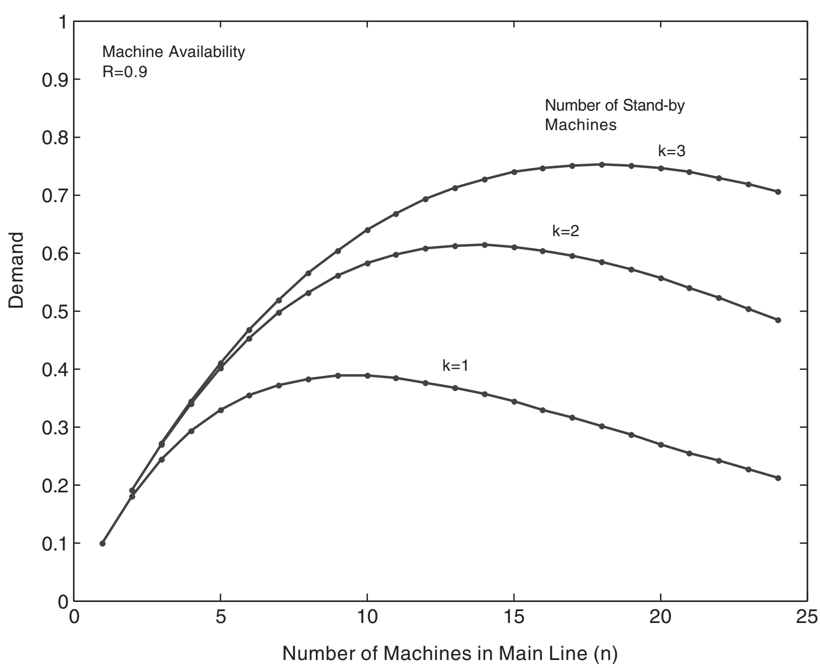


Figure 6. Serial production line demand for standby machines.

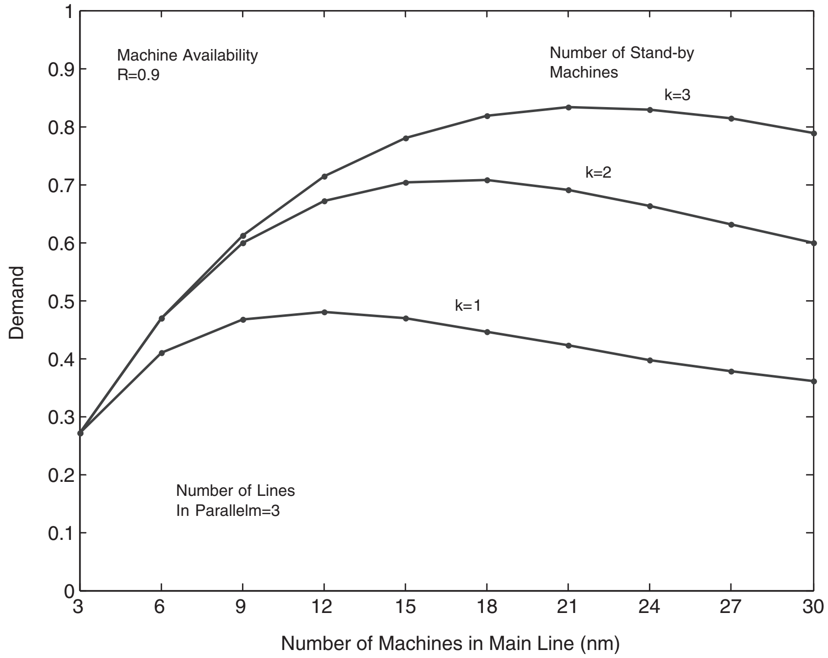


Figure 7. Parallel-serial production line demand for standby machines.

than in the pure serial system. Note that the difference between the lines plotted in figure 6 and 7 is the relative contribution each standby machine can provide toward the demand. Figures 6 and 7 also show, as expected, for both a pure serial line and a parallel-serial line, the greater the length of the main line, the more standby machines that can be used to achieve productivity gains. Machine availability also influences the saturation point of the main line. As can be seen in figure 8, plotting equations (8) and (9), the more available a serial main line machine is, the greater its length may be before saturation occurs in one or two standby machines.

Just as buffers are used to isolate machine failure from system failure in asynchronous systems, providing reserve capacity to a synchronized system also isolates machine failure from system failure. As such, it is possible to determine a buffer size that provides the same productivity gain for each standby machine in a reserve capacity system. Table 1 lists the equivalent average buffer size necessary to achieve the same productivity for a pure serial line of 12 machines (11 buffers). Buffer size was calculated using an aggregation/decomposition technique according to Gershwin (1987) and Dallery *et al.* (1989). This technique, incorporated by Yang *et al.* (1999) into a prototype software program, PAMS, provides the optimum buffer allocation to achieve a given level of productivity. Program inputs are production rate, MTTF and MTTR of each station in the production line. The program output provides the total number of buffer spaces to meet a given productivity level, and their location within the line. Table 1 shows the average buffer size over the 11 buffers. Unfortunately, this software limits the maximum buffer space (sum of the size of all of the buffers) to 2000 units.

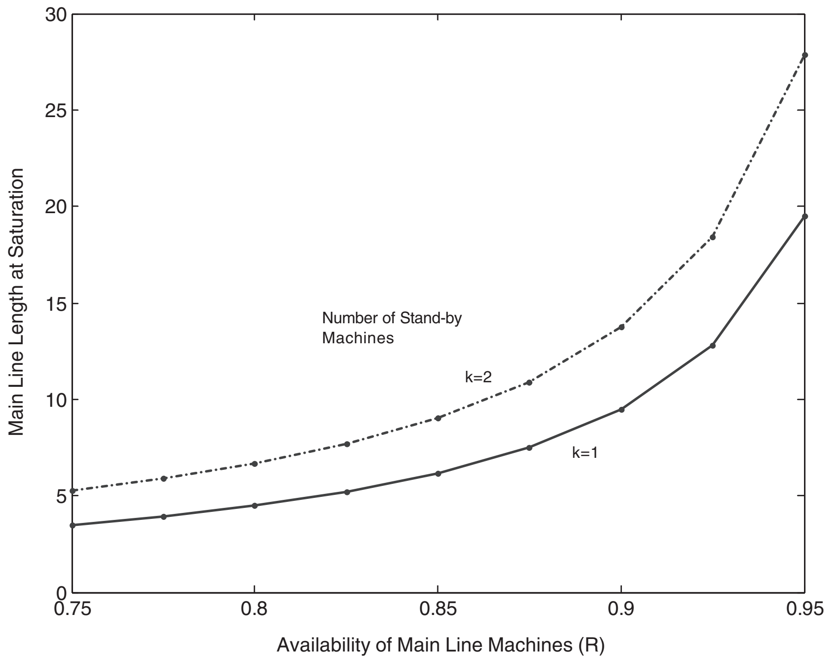


Figure 8. Machine availability and the influence on main line length for saturation.

| Number of standby machines | Reserve capacity productivity estimate | PAMS optimal average buffer size (MTTR 30) | PAMS optimal average buffer size (MTTR 120) |
|---|---|---|---|
| 0 | 0.282 | 0 | 0 |
| Min buffer size | 0.475/0.436 | 1 | 1 |
| 1 | 0.557 | 6 | 149.1 |
| 2 | 0.745 | 37.3 | $>182$ |
| 3 | 0.847 | 151.7 | |
| 4 | 0.895 | $>182$ | |
| 5 | 0.916 | | |

Pure serial line of length $n = 12$, number of buffers = 11, machine availability $R = 0.9$, $R_s = 0.81$, $\mu_m = 1$, $\mu_s = 0.9$. MTTR, mean time to repair.

Table 1. Equivalent average buffer size of serial production line with reserve capacity.

To achieve productivity of the system on the same order as the availability of its constituent machines, table 1 shows that it is necessary to provide large buffers. In fact, even with extremely large buffers, it is not possible to achieve productivity levels higher than the availability of the constituent machines. A reserve capacity system will permit higher than expected productivity levels than the availability of main line constituent machines, as it is equivalent to a system in parallel. Further, the necessary size of buffers for a given level of productivity grows as the MTTR of the machines increases. Since buffer cost is generally lower than the investment cost of a standby machine, it is necessary to determine the trade-off between the choices.

Beyond the replacement of buffers, reserve capacity can also be applied for other production line design decisions. Under the paradigm of reconfigurable manufacturing, in meeting market demand, it is desirable to provide the correct level of production at the time that it is needed. In traditional approaches to production line design, production can only be scaled by the smallest set of production units that can complete a product. That is, a serial transfer line must be duplicated in full to increase output, in this case by double, while the addition of one machine to a set of $m$ parallel flexible machines performing all production tasks will increase output by $1/m$. The addition of reserve capacity allows for an incremental increase in productivity of machining lines, with line expansion complete when the number of standby machines equals the main line length. For example, in the 12 machine pure serial line of table 1, adding a second standby machine increases the production rate of the system by 33%. Of course, as noted above, there is a diminishing return for each standby machine, so the addition of the 12th machine will provide a much smaller increase to the productivity. Thus, it may be warranted to invest in several machines to complete a parallel-serial line as the number of standby machines becomes large.

The equations developed for reserve capacity can also be applied to the analysis of serial assembly lines. The Toyoda production system is designed to address quality concerns by allowing the workers the autonomy to stop the production line. Typically, this consists of a two-step process; first, a warning signal is given where an assembly line worker indicates they need help resolving a problem. While a team leader helps to resolve the problem, the line continues to move. If the problem continues beyond a point on the line, the line is halted until it can be resolved. To avoid a single problem shutting down the complete assembly line, the automated material transfer system is segmented. The reserve capacity equations can be used to model the assembly line to provide the most appropriate line segment length to achieve a given level of productivity, where the team leader is the 'standby machine'.

## 4. Conclusions

Models have been developed to predict the productivity of pure serial and parallel-serial systems with reserve capacity. These models provide new opportunities for system designers to consider bufferless systems. The minimal exploration of other system configurations beyond buffers in decoupling machine and system failure is primarily due to the history of manufacturing. For mass production, high productivity dictated the installation of dedicated serial manufacturing lines, resulting in buffer decoupling as the most economic method of failure mitigation. However, with flexible and reconfigurable manufacturing system, this paradigm is called into question. The total system cost may not be a minimum with the exclusive use of buffers. Depending on system requirements, standby machines with or without buffers may produce a better cost/productivity optimum. This may also depend on other life cycle cost issues as scalability of operations, long-term variability to product demand and, in reconfigurable manufacturing systems, the possibility of transfer of production assets from one product family to another.

Obviously, significant gains to productivity are obtained by the addition of reserve capacity. However, there is added investment in multicapable standby machines and material-handling equipment. It is possible to reduce material handling by providing flexibility in the manufacturing configuration. Despite this possibility, every system must be analysed to determine if the productivity gains and other benefits of a reserve capacity system are significant enough over the relative inexpense of a buffer.

To understand reserve capacity systems further and to investigate the trade-offs between standby machines and buffers, future work will address the influence of material transfer on productivity, hybrid systems of buffers and standby machines, the use of multicapability standby machines in continuous production (as opposed to solely in reserve), and the influence of failure and reconfiguration frequency.

## Appendix

The following specific example illustrates the application of equations (10–14) in calculating the productivity of a synchronized pure serial line, where every station has its own availability. This line has five stations ($n = 5$) and two standby machines ($k = 2$), and the index set of machines is $\bar{a} = (1, 2, 3, 4, 5)$. The system demand given by equation (10) is given by the combination of the probability of occurrence of one (first half of the expression) or two (second half of the expression) failures in the main line:

$$D_2 = \sum_{a_1=1}^{5}\left[(1 - R_{a_1})\prod_{j\notin\bar{a}} R_j\right] + \sum_{a_1=1}^{4}\sum_{a_2=a_1+1}^{5}\left[(1 - R_{a_1})(1 - R_{a_2})\prod_{j\notin\bar{a}} R_j\right] \tag{A.1}$$

In the second half of the expression, the first upper summation limit is 4, and the second lower summation limit $a_1 + 1$. This reflects that two failures are occurring, and are not the same machine.

To determine the probability that there are sufficient standby machines functioning to cover the main line machine failures, a generating function is used to expand out all possible combinations of failures, assuming that the standby machine's availability is directly related to the operation it is covering (equation 11). For $i = 1$ and 2, representing the number of failed machines in the main line:

$$\begin{aligned} gf_{i=1} &= \sum_{q=0}^{1}(1 - R_{a_p})^q x^q = 1 + (1 - R_{a_1})x \\ gf_{i=2} &= \left(\sum_{q=0}^{0}(1 - R_{a_p})^q x^q\right)\left(\sum_{q=0}^{0}(1 - R_{a_p})^q x^q\right) = 1. \end{aligned} \tag{A.2}$$

Therefore, $b_0 = 1$, for $i = 1$ and 2, and $b_1 = (1 - R_{a_1})$ for $i = 2$. Applying this to equation (12):

$$\begin{aligned} R'_{s0} &= 1 \\ R'_{s1} &= R_{a_1}\big(1 + (1 - R_{a_1})\big) = R_{a_1} + R_{a_1}(1 - R_{a_1}) \\ R'_{s2} &= R_{a_1}R_{a_2}. \end{aligned} \tag{A.3}$$

This formulation gives the probability that when one standby machine is required, one or more standby machines are available, and when two standby machines are required, two are available.

From equation (13), productivity is:

$$P_{\text{reserve}} = \prod_{i=1}^{5} R_i + \left(1 - \frac{1}{5}(1-\alpha)\right) \sum_{a_1=1}^{5} \left[ (1 - R_{a_1}) \left( \prod_{j \notin \vec{a}} R_j \right) \left(2R_{a_1} - R_{a_1}^2\right) \right]$$

$$+ \left(1 - \frac{2}{5}(1-\alpha)\right) \sum_{a_1=1}^{4} \sum_{a_2=a_1+1}^{5} \left[ (1 - R_{a_1})(1 - R_{a_{2i}}) \left( \prod_{j \notin \vec{a}} R_j \right) R_{a_1} R_{a_2} \right]. \quad \text{(A.4)}$$

The first term represents the system state probability when all machines are functioning. The second term represents the system state probability when any one machine has failed, and can be backed up by at least one standby machine. The third term represents the system state probability where two machines have failed, and both standby machines must be functional for the system to continue to produce.

When applying the standby machine discounting factor $\beta$ of equation (14), the generating functions and probability of sufficient standby machines becomes:

$$\begin{aligned} gf_{i=1} &= \sum_{q=0}^{1} (1 - \beta_{a_p} R_{a_p})^q x^q = 1 + (1 - \beta_{a_p} R_{a_1}) x \\ gf_{i=2} &= \left( \sum_{q=0}^{0} (1 - \beta_{a_p} R_{a_p})^q x^q \right) \left( \sum_{q=0}^{0} (1 - \beta_{a_p} R_{a_p})^q x^q \right) = 1 \end{aligned} \quad \text{(A.5)}$$

giving, $b_0 = 1$, for $i = 1$ and 2, and $b_1 = (1 - \beta_{a_1} R_{a_1})$ for $i = 2$, and

$$\begin{aligned} R'_{s0} &= 1 \\ R'_{s1} &= \beta_{a_1} R_{a_1} \left(1 + (1 - \beta_{a_1} R_{a_1})\right) = \beta_{a_1} R_{a_1} + \beta_{a_1} R_{a_1} (1 - \beta_{a_1} R_{a_1}) \\ R'_{s2} &= \beta_{a_1} R_{a_1} \beta_{a_2} R_{a_2}. \end{aligned} \quad \text{(A.6)}$$

**Acknowledgements**

The authors gratefully acknowledge the financial support of the NSF Engineering Research Center for Reconfigurable Machining Systems (NSF Grant EEC95-92125), University of Michigan, and the valuable input from the center's industrial partners. Professor M. Shpitalni was on sabbatical leave from the Technion, Israel.

## References

Dallery, Y., David, R. and Xie, X.-L., 1989, Approximate analysis of transfer lines with unreliable machines and finite buffers. *IEEE Transactions on Automatic Control*, **34**, 943–953.

Freiheit, T., Shpitalni, M. and Hu, S. J., 2002, Productivity of parallel-serial manufacturing lines with and without crossover. *Proceedings of the 2002 ASME International Mechanical Engineering Congress and Exposition*, 17–22 November 2002, New Orleans, LA.

Galikowsky, C., Sivazlian, B. D. and Chaovalitwongse, P., 1996, Optimal redundancies for reliability and availability of series systems. *Microelectronic Reliability*, **36**, 1537–1546.

Gershwin, S. B., 1987, An efficient decomposition method for the approximate evaluation of tandem queues with finite storage space and blocking. *Operations Research*, **35**, 291–305.

Gershwin, S. B. and Berman, O., 1981, Analysis of transfer lines consisting of two unreliable machines with random processing times and finite storage buffers. *AIIE Transactions*, **13**, 2–11.

Hopp, W. and Spearman, M., 1996, *Factory Physics: Foundations of Manufacturing Management* (Boston: Irwin McGraw-Hill).

Koren, Y., Hu, S. J. and Weber, T., 1998, Impact of manufacturing system configuration on performance. *Annals of the CIRP*, **47**, 369–372.

Kubat, P. and Sumita, U., 1985, Buffers and backup machines in automatic transfer lines. *International Journal of Production Research*, **23**, 1259–1270.

Kumar, A. and Agarwal, M., 1980, A review of standby redundant systems. *IEEE Transactions of Reliability*, **R-29**, 290–294.

Meng, F. C., 1995, Comparing the MTBF of four systems with standby components. *Microelectronic Reliability*, **35**, 1031–1035.

Shen, K. and Xie, M., 1991, The effectiveness of adding standby redundancy at system and component levels, *IEEE Transactions on Reliability*, **40**, 53–55.

Wang, W. and Loman, J., 2002, Reliability/availability of $k$-out-of-$n$ system with $m$ cold standby units. *2002 Proceedings of the Annual Reliability and Maintainability Symposium*, IEEE 02ch37318, pp. 450–455.

Yang, S., Hu, S. J. and Koren, Y., 1999, Approximate analysis of production lines with loop, scrapping, and serial/parallel blocks. *Second Aegean International Conference on* '*Analysis and Modeling of Manufacturing Systems*', 16–20 May, Tinos Island, Greece.