**O. Masory**
Mechanical Engineering Department,
Texas A&M University,
College Station, Texas 77843

**Y. Koren**
Faculty of Mechanical Engineering,
Technion—Israel Institute
of Technology,
Haifa 32000, Israel

# Stability Analysis of a Constant Force Adaptive Control System for Turning

*Adaptive Control Constrains (ACC) as applied to machine tool systems refers to the regulation of the cutting process characteristics such as spindle power, cutting force etc., by varying the only changeable process parameters which are feedrate and cutting speed. Even a simple ACC system, as in this case, in which the cutting force is regulated by changing the feedrate, is quite complex. Experiments with this simple system, implemented on a computerized numerical control (CNC) lathe, have shown that the CNC/ACC system can become unstable due to changes in depth-of-cut or spindle speed. This paper presents stability analysis of the above CNC/ACC system, thus making it possible to select appropriate gains which will maintain the system stable. Also, experimental results, which describe the stability region of such a system, will be provided.*

## 1 Introduction

The availability of a dedicated computer in the control system and the need for higher productivity has greatly accelerated the development of adaptive control (AC) systems, which are based on automatic control of the operating parameters with reference to measurements of the machining process characteristics. AC systems for machine tools can be classified into two categories: (1) Adaptive Control Optimization (ACO), and (2) Adaptive Control Constraints (ACC). ACO refers to systems in which a given index of performance (usually an economic function) is extremized subject to process constraints. With ACC the operating parameters are maximized such that the system operates on a specified constraint, which arises due to the process physics, the machine tool performance, etc.

Although there has been considerable research on the development of ACO systems [1–5], hardly if any, of these systems are used in practice. The major problems with such systems have been difficulties in defining realistic indices of performance and the lack of suitable sensors which reliably measure on line the necessary parameters in the production environment. Practically all the AC systems which are used in production today [6–11] are of the ACC type and seldom involve the control of more than one operating parameter [12]. The main research efforts in ACC systems have been devoted to the problem of eliminating chatter [13, 14] and to the determination of programmed constraints, such as cutting power [15], or force [16, 17].

Similarly the main objective of the ACC system that this paper is concerned with is to increase the productivity of a machine tool by applying the highest feedrates that are compatible with a maximum allowable cutting force, thus eliminating tool breakage. Our experimental work demonstrated that a proper selection of the adaptive controller gain is very critical if wide variation in depth-of-cut and spindle speed are permitted in the system [18, 19]. The reason is that in an ACC system the process itself is part of the control loop, and therefore, variations in the process, such as spindle speed and depth-of-cut, directly affect the control parameters of the loop, and consequently the AC system might become unstable [17, 18].

Instability in the AC systems is not a familiar phenomenon to people on the shop floor, since most of them do not use AC systems in production. Users of AC systems encounter this instability condition rather infrequently in practice, since their parts programmers are experienced enough to avoid large changes in depth-of-cut or spindle speed. This, however, means that the production rate is decreased and that the objective of the AC system is not fully achieved.

The sensitivity of the AC controller performance to the selection of the open loop gain calls for a different approach to AC system design: The system should operate with a variable open loop gain which adapts itself according to the changes in the cutting process. The necessity for this type of approach has been mentioned in [20] and has been explicitly stated by Stute [21] in the report of the American Machine Tool Task Force. Also Mathias [11] states that Macotech Corporation's commercial MACXX-C system has a simple variable control gain algorithm which "decreases the control loop gain at the onset of any feedrate oscillation" to avoid AC control loop instability, but he does not propose a systematic method to adjust the gain.

However, in order to be able to vary the AC open-loop gain during cutting it is necessary to find out the stability region of the system. This paper presents stability analysis of a constant force ACC system [18–20] for turning, by which the stability

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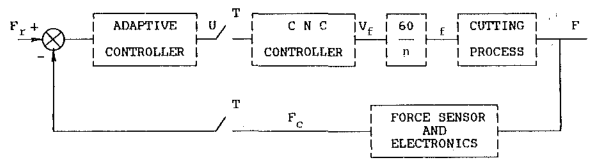


Fig. 1 Force controller block diagram

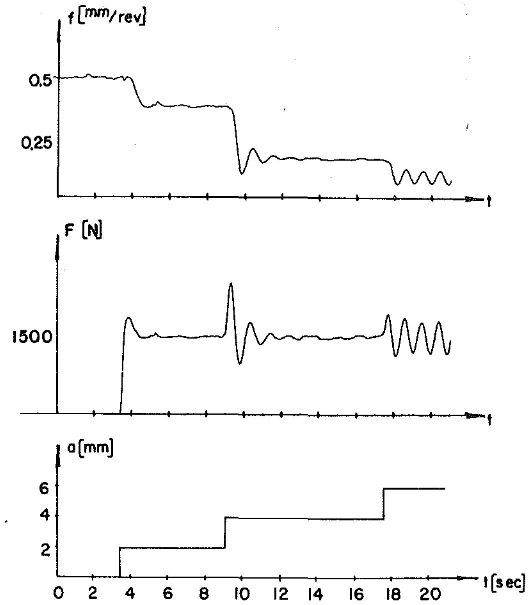


Fig. 2 Conventional AC system response to changes in depth-of-cut

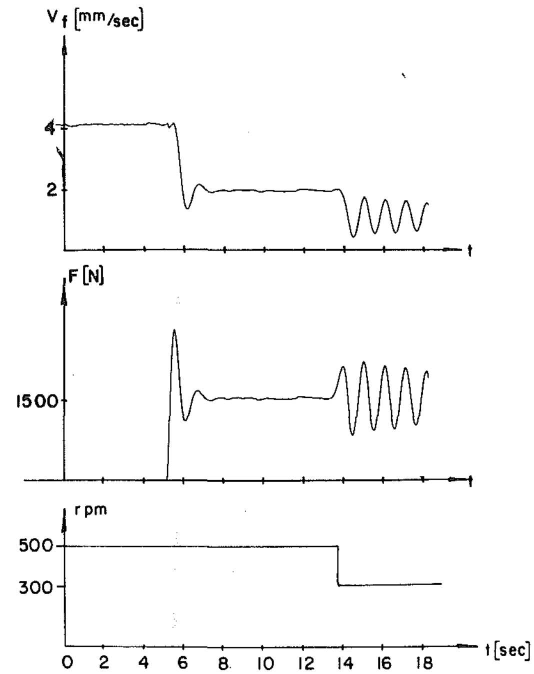


Fig. 3 Conventional AC system response to changes in spindle speed

region of the system can be determined. Also, experimental results describing the stability region of the system will be provided.

## 2 The Adaptive Control Loop

The basic block diagram of the ACC system is shown in Fig. 1. It is a sampled data feedback loop where the feed $f$ adapts itself to the actual cutting force $F$ and varies according to changes in machining conditions as cutting proceeds. The actual main force is sampled every $T$ seconds, converted to a digital signal $F_c$, and is immediately compared with a predetermined allowable reference force $F_r$. The difference between $F_r$ and $F_c$ is the force error $E$, which is used as an input to the ACC controller. In turn, the ACC controller sends a correction signal to the feedrate routine contained in the CNC control program, which changes the feedrate, or axis velocity, reference of the CNC servo system. As result, the feedrate $V_f$ varies and consequently the cutting force $F$.

In order to completely eliminate the force error, the ACC controller output command $U$ should be proportional to the time integral of the force error. The simplest algorithm for such an integral policy can be written:

$$U(i) = K_c T \Sigma E(i) \tag{1}$$

where $K_c$ is a constant denoting as the ACC controller gain, and $E(i)$ is the force error at the $i$th sampling period, which is given by:

$$E(i) = F_r(i) - F_c(i) \tag{2}$$

As long as there is an error, the command $U$ varies the machine feedrate in a direction to correct this error. At steady-state, however, the error in the force is zero, causing the condition $U(i) = U(i-1)$, which means that the feedrate command $U(i)$ is constant, maintaining the actual force equal to the required one.

The CNC servo, of each axis, has the characteristic of second order lag system [22], and therefore the relationship between the feedrate $V_f$ and the command $U$ is given by:

$$\frac{V_f(s)}{U(s)} = \frac{K_n \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{3}$$

where

$K_n$—is the CNC servo gain
$\omega_n$—is the natural frequency of the servo
$\xi$—is the damping coefficient of the servo
$s$—Laplace variable

The machining feed is given by:

$$f = (60/n) V_f \tag{4}$$

where $n$ is the spindle speed in rpm.

The cutting force $F$, in steady state, is a function of the feed and the depth-of-cut $a$, and can be approximated by:

$$F = K_s a f^p \tag{5}$$

where $K_s$ is the specific cutting force and $p$ is a constant ($p < 1$), both depending on the workpiece and tool materials. Assuming that $p = 1$ and that the process dynamics can be described by a transfer function $G_p(s)$, the relationship between the cutting force and the feed is given by:

$$\frac{F(s)}{f(s)} = K_s a G_p(s) \tag{6}$$

The cutting force $F$ is measured by a force sensor and then converted to a digital value $F_c$. The conversion factor between $F$ and $F_c$, including the sensor electronics, is $K_e$:

$$F_c = K_e F \tag{7}$$

Combining equations (1) through (7) yields the continuous part of the open-loop transfer function of the ACC system:

$$G(s) = \frac{K\omega_n^2}{(s^2 + 2\xi\omega_n s + \omega_n^2)} G_p(s) \tag{8}$$

where $K$, the open-loop gain, is defined by:

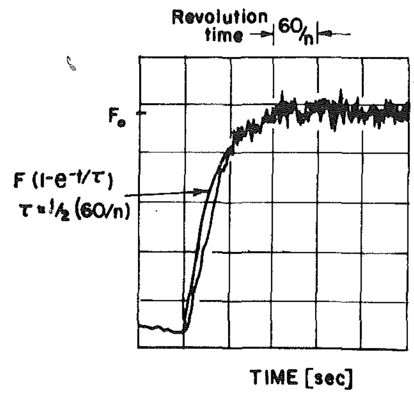

Fig. 4 Cutting force response during tool engagement

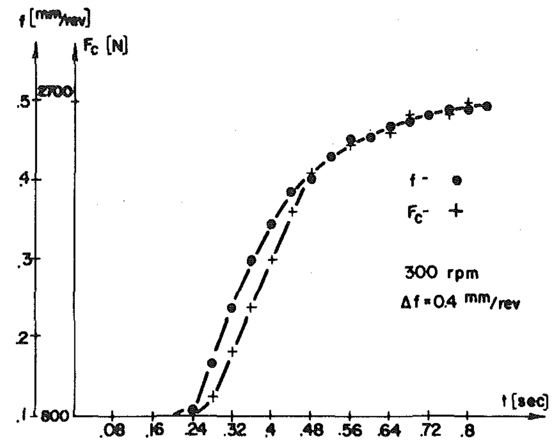

Fig. 5 Cutting force lag during change in feedrate

$$K=(60/n)K_c TK_n K_e K_s a \tag{9}$$

At this point it can be realized that since the depth-of-cut ($a$) and the spindle speed ($n$) are contained in the open loop gain $K$, the selection of the controller gain $K_c$, which is the only adjustable gain in the system, is critical to the proper operation of the ACC system.

The results of two experiments demonstrating the stability problem, due to changes in depth-of-cut and spindle speed, are given in Figs. 2 and 3. In these experiments the adaptive controller gain was $K_c = 0.5$ and the sampling period $T = 0.1$ s. The depth-of-cut is increased by increments of 2 mm and the system is stable as long as the depth-of-cut does not exceed 4 mm. At depth-of-cut of 6 mm the system becomes unstable. Furthermore, when running the system with constant depth-of-cut and different spindle speeds, the system became unstable with lower spindle speed, as shown in Fig. 3.

## 3 Process Modeling

In order to be able to model the dynamic behavior of the cutting force, in terms of control theory, one has to observe the cutting force transient response in the following three cases:

(*a*) Cutting force response due to a step change in depth-of-cut or due to the engagement of the tool with the workpiece (at constant feed and spindle speed).
(*b*) Cutting force response due to a step change in the feedrate (at constant depth-of-cut and spindle speed).
(*c*) Cutting force response due to a step change in spindle speed (at constant depth-of-cut and feedrate).

In all these cases, the cross-section of the removed chip is linearly increased during one revolution of the workpiece, starting when the change occurs. At the end of this revolution, the chip cross-section area reaches its maximum value and from this point it remains constant. Since the cutting force is proportional to the chip cross-section area, (assuming $p=1$ in equation (5)) it is expected to observe a transient in the cutting force when these phenomena take place. The equations describing the changes in the chip cross-section area for these cases are given in the Appendix A.

Figure 4 [23-26] describes measurement of the cutting force during the engagement of the tool with the workpiece. As expected, the force increases linearly during the first revolution of the workpiece and reaches its maximum value at the end of this revolution. The deviation from the straight line is due to elasticity of the tool holder etc. Same transient is expected for a step change in depth-of-cut.

To observe the lag between the cutting force and the feedrate, the following experiment was carried out. During

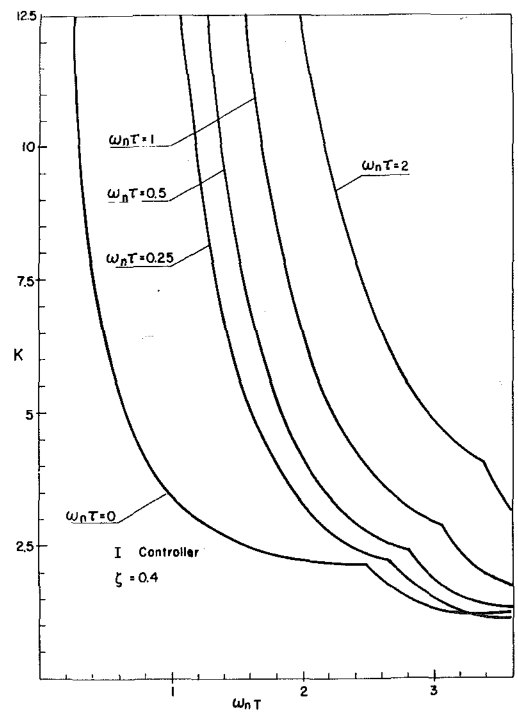

Fig. 6 Stability region of the ACC system

cutting with a constant depth-of-cut and spindle speed, the feedrate reference for the CNC servo was stepped from 0.5 mm/s to 2.5 mm/s (which corresponds to 0.1 mm/rev to 0.5 mm/rev at spindle speed of 300 rpm) and both the feed and the cutting force were recorded. These results were normalized and redrawn on the same chart, as described in Fig. 5, which indicates clearly, as expected, that the cutting force lags after the feedrate.

The force transient can be approximated (see Fig. 4) by:

$$F(t)=F(1-e^{-(t/\tau)}) \tag{10}$$

where $F$ is the cutting force steady-state value given by equation (5), and $\tau$ is a time constant equals to time of half revolution of the spindle:

$$\tau=30/n \tag{11}$$

Since changes in depth-of-cut and spindle speed affect the magnitude of the cutting force, resulting changes in the feed in order to keep the cutting force constant, the dynamic response of the cutting force to these changes can be described by first order lag between the force and the feed:

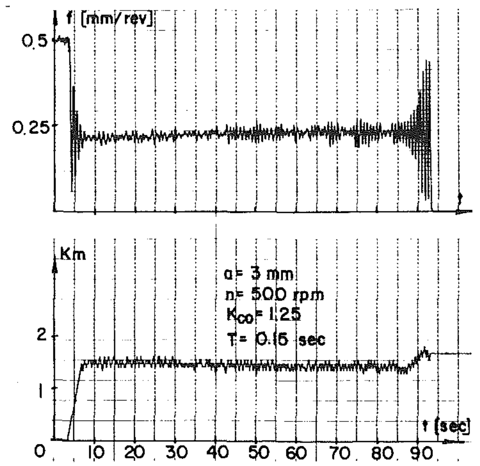


**Fig. 7 Typical experiment for determination the stability region of the system**

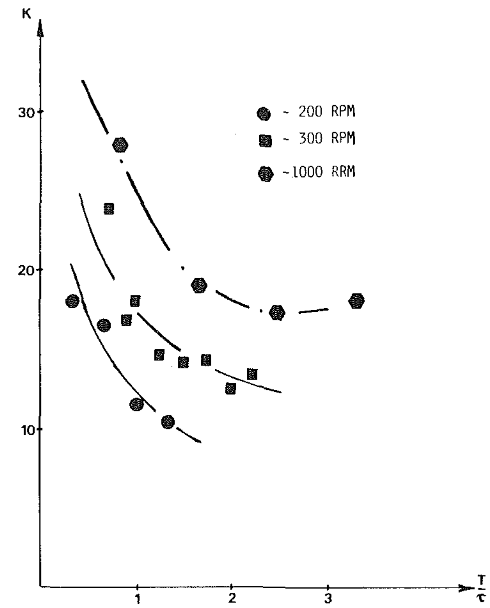


***Fig. 8 The influence of process time constant on stability***

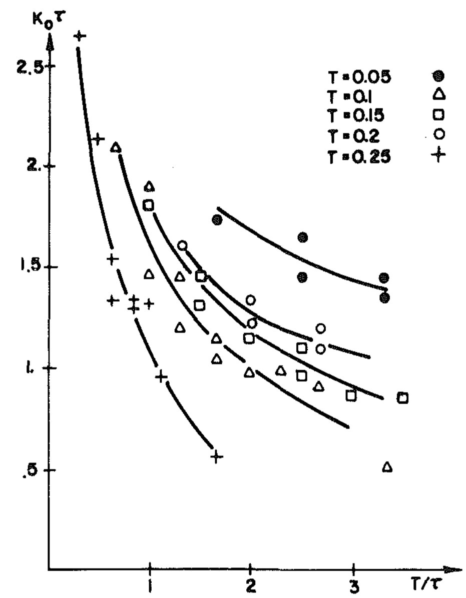


**Fig. 9 The influence of sampling time on stability**

$$\frac{F(s)}{f(s)} = \frac{1}{\tau s+1} = G_p(s) \tag{12}$$

Substituting equation (12) into equation (10) yields:

$$G(s) = K\frac{\omega_n{}^2}{(s^2+2\xi\omega_n s+\omega_n{}^2)(\tau s+1)} \tag{13}$$

where $K$ is the open-loop gain given by equation (9).

## Stability Analysis

The ACC system is a sampled-data system and therefore it should be analyzed by using $Z$ transform technique [27]. The open-loop transfer function is given in $z$ domain by:

$$G(z) = K\frac{1}{z-1}Z\left|\frac{1-e^{-sT}}{s}\ \frac{}{s(s^2+2\xi\omega_n s+\omega_n{}^2)(\tau s+1)}\right| \tag{14}$$

Partial-fraction expansion of equation (14) and using $z$ transform tables yields:

$$G(z) = K\left|\frac{z}{z-1} + B_1\frac{z}{z-E_2} + B_2\frac{z(z-h)}{z+2hz+E_1} + B_3\frac{z}{z+2hz+E_1}\right| \tag{15}$$

where $B_1$, $B_2$, and $B_3$ are constants depends on $T$, $\tau$, $\xi$, and $\omega_n$.

$h = \cos(\omega_\psi\sqrt{1-\xi^2}\ T)\exp(-\xi\omega_n T)$
$E_1 = \exp(-2\xi\omega_n T)$
$E_2 = \exp(-T/\tau)$
$z = Z$ transform variable.

The characteristic equation of the system is given by:

$$1+G(z)=0 \tag{16}$$

which is a polynomial of fourth degree.

The stability region of the system was determined by Schur-Cohen criterion and is shown in Fig. 6. As expected, the allowed open-loop gain of the system decreases as the process time constant decreases and increases as the sampling frequency increases. Furthermore, the foregoing ACC system has "self-stabilizing" characteristic: As the process time constant decreases, due to increase of spindle speed, the open-loop $K$ gain is automatically decreased since it is proportional to $1/n$ (equation (9)). It should be emphasized that the results presented in Fig. 6 are not accurate due to the assumption $p = 1$ in equation (5).

## 5 Experiments Results

In order to obtain better results on the stability region of the above system, the following experiment was repeated for different sampling times $T$ and spindle speeds $n$. At constant depth-of-cut, spindle speed and sampling time the ACC controller gain $K_c$ was incremented by a known value every 10 seconds until the system became unstable. At this point the value of $K_c$ was registered and therefore the open-loop gain of stability threshold could be calculated by equation (9). However, this calculation procedure might provide unaccurate results due to unknown changes in the value of the specific cutting force $K_s$, and due to the assumption $p=1$ in equation (5). Therefore it is necessary to estimate the value of the open loop gain of the ACC system in real-time during the cutting process [28].

Since the gain of the ACC controller $K_c$ is known (see equation (9)), only the rest components of the open loop gain have to be estimated. For this purpose, an estimation algorithm, which was executes in real-time, was added to the control program. This algorithm estimates the value of the rest components of the open loop gain, denoted as $K_m$. At this point the open-loop gain $K$ can be calculated by:

$$K = \frac{K_c K_m}{T} \tag{17}$$

In order to check the estimation results, the values of $K$ and $p$ were calculated using the values of $K_m$. These values were found to be very accurate, within a few percentages, compared with experimental values given in literature [29, 30].

Figure 7 shows the results of a typical experiment which was taken with a spindle speed of 500 rpm, depth-of-cut of 3 mm, and sampling time of 0.15 s. The ACC controller gain was set to initial value of 1.25 and was incremented by 1/16 every 10 seconds. The system became unstable at $K_c$ = 1.75, and at that point the estimated value for the rest components of the open-loop gain, $K$, was 1.5.

The results of these experiments are shown in Figs. 8 and 9. Figure 8 describes the influence of the process time constant on the stability region of the system. As expected, as the time constant increases the allowed open-loop gain decreases. Figure 9 emphasizes the influence of the sampling period $T$ on the stability of the system. Again, as the sampling time decreases the allowed gain increases.

It should be mentioned that the actual stability region is smaller than the calculated one (Fig. 6) due to the assumption $p = 1$ in equation (5). In the above experiments the feed ranges between 0.05 to 0.3 mm/rev and therefore the term $f^p$ (equation (5)) is greater than $f$ meaning that the actual gain is greater than the one was assumed.

## 6 Conclusion

It has been shown that in adaptive control constraints systems for machining, the cutting process itself affects the open loop gain of the system. Therefore system stability is very sensitive to variations in the cutting process. This problem might occur in ACC and ACO systems since in most cases the constrains (such as forces, torques and spindle power) depend on the control variables: feed and the cutting speed. Thus it is necessary to find the stability conditions of the system in order to select appropriate gains.

It should be emphasized that a selection of a very low open loop gain, which insures stability in case of wide variations in the process, would not solve the problem. Low gain will result in very sluggish transient response which might cause damage to the tool. Therefore it is recommended to add another adaption loop which changes the ACC controller gain in such a way that the open loop gain obtains a constant reference value. This value will be selected according to the stability conditions of the system, as described in this paper, and the required transient response.

## Acknowledgment

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## References

1 Centner, R., "Final Report on Development of Active Control Technique for Numerically Controlled Milling Machine," Technical Documentary, Report ML-TDR-64-279, Aug. 1964.

2 Mathias, A., "An Effective System for Adaptive Control of the Milling Process," SME Paper MS-68-202, 1968.

3 Colwell, L. W., Frederick, J. R., and Quachenbush, L. J., "Research in Support of Numerical and Adaptive Control in Manufacturing," The University of Michigan, Ann Arbor, 1969.

4 Beadle, B. R., and Bollinger, J. G., "Computer Adaptive Control of Machine Tools," *Annals of the CIRP*, 1970.

5 Bedini, R., Lisini, G. G., and Pinotti, P. C., "Experiments on Adaptive Control of a Milling Machine," ASME Journal of Engineering for Industry, Vol. 98, Feb. 1976, pp. 239-245.

6 Mathias, A., "Adaptive Controlled Profile Milling," SME Tech. Paper MS-70-5 63, 1970.

7 Groover, Mickell, P., "A Definition and Survey of Adaptive Control Machining," SME Tech. Paper MS-70-561, 1970.

8 Frecka, Thomas, R., "Torque Controlled Drilling," IEEE-IAS Machine Tools Industry Conf., Oct. 1975.

9 Mathias, A., "New Developments in Adaptive Control of Milling Machines," IEEE-IAS Machine Tools Industry Conf., Milwaukee, Wis., Oct. 28-30, 1975.

10 Mathias, A., "An Adaptive Controlled Milling Machine," SME Tech. Paper MS-76-260, 1976.

11 Mathias, A., "Software Adaptive Control — Optimum Productivity for CAM," SME Tech. Paper MS-77-252, 1977.

12 Beadle, B. R., and Bollinger, J. G., "A Constrained Search Adaptive Controller for Metal Cutting," *Proc. of the 2nd NAMRC Conf.*, University of Wisconsin, Madison, May 1974, pp. 273-280.

13 Weck, M., Verhaag, E., and Gather, M., "Adaptive Control for Face-Milling Operations with Strategies for Avoiding Chatter Vibrations and for Automatic Cut Distribution," *Annals of the CIRP*, Vol. 24, 1975, pp. 405-409.

14 Inamura, T., Senda, T., and Sata, T., "Computer Control of Chattering in Turning Operation," *Annals of the CIRP*, Vol. 26, 1977, pp. 181-186.

15 DeFilippi, A., and Ippolito, R., "The Influence of Constraints on Cutting Condition Optimization," *Annals of the CIRP*, Vol. 24, 1975, pp. 417-421.

16 Tlusty, J., and Elbestawi, M., "Constraints in Adaptive Control with Flexiable End Milling," *Annals of the CIRP*, Vol. 28, 1979, pp. 253-255.

17 Tlusty, J., and Elbastawi, M., "Analysis of Transient in an Adaptive Control Servomechanism for Milling with Constant Force," ASME Journal of Engineering for Industry, Vol. 99, No. 3, Aug. 1977.

18 Masory, O., and Koren, Y., "Adaptive Control System for Turning," *Annals of the CIRP*, Vol. 29, No. 1, 1980.

19 Koren, Y., and Masory, O., "Adaptive Control with Process Estimation," *Annals of the CIRP*, Vol. 30, No. 1, 1981.

20 Masory, O., and Koren, Y., "Variable — Gain Adaptive Control for Turning," *J. of Manufacturing Systems*, Vol. 2, No. 2, 1983.

21 Stute, G., "Adaptive Control," *Proc. of the Machine Tool Task Force Conf.*, Vol. 4, Chapter 14, Oct. 1980.

22 Koren, Y., "Design of a Digital Loop for Numerical Control," *IEEE Trans. on Industrial Electronics and Control Instrumentation*, Vol. IECI-25, No. 3, Aug. 1983.

23 Colwell, L. V., "Tracking Tool Deterioration by Computer (During Actual Machining)," *Annals of the CIRP*, Vol. 23, No. 1, 1974.

24 Colwell, L. V., Mazur, J. C., and DeVries, W. R., "Analytical Strategies for Automatic Tracking of Tool Wear," *Proc. NAMRC Conf.*, No. 6, 1978.

25 Colwell, L. V., and Mazur, J. C., "Tool Wear Tracking in the Production Shop," *Proc. NAMRC Conf.*, No. 4, 1976.

26 Colwell, L. V., and Mazur, J. C., "Real Time Computer Diagnostic (A Research Tool for Metal Cutting)," *Annals of the CIRP*, Vol. 28, No. 1, 1979.

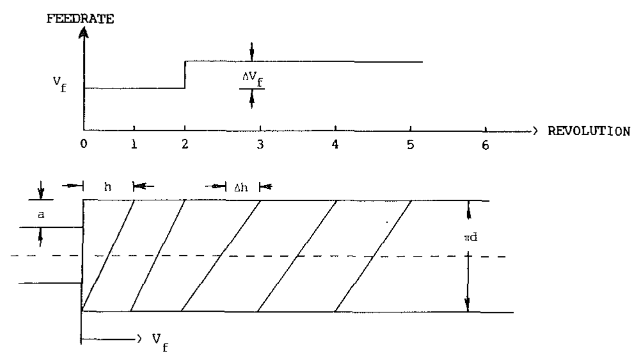


**Fig. A.1 Chip width geometry during step change in depth-of-cut**

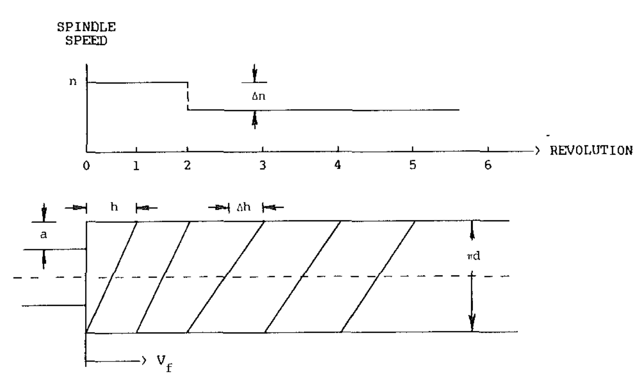


**Fig. A.2 Chip width geometry during step change in feed**

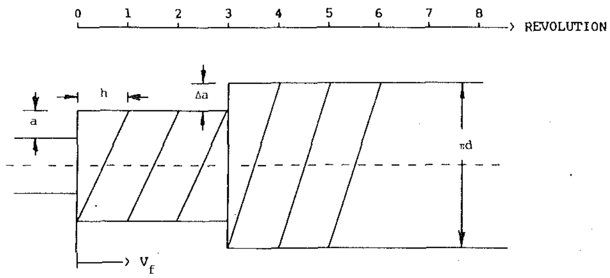


**Fig. A.3 Chip width geometry during step change in spindle speed**

27 Jury, E. I., *Sampled-Data Control Systems*, Robert E. Krieger Publishing Company, Inc., 1977.
28 Masory, O., "Real-Time Estimation of Cutting Process Parameters in Turning," ASME Journal of Engineering for Industry, Vol. 106, No. 3, Aug. 1983, pp. 218–221.
29 Kronenberg, M., *Machining Science and Application*, Pergamon Press, 1966.
30 Koren, Y., "Dynamic and Static Optimization of the Cutting Process," *Proc. NAMRC*, 1973.

## APPENDIX A

### Chip Cross-section Area Geometry

The cross-section area, $A$, of the removed chip for the three cases is given below.

case ($a$): Change in depth-of-cut (Fig. A.1)

$$A(t) = (a + \Delta a)h(t) \tag{A.1}$$

where $a$ is the initial depth-of-cut and $h(t)$ is the chip width given by:

$$h(t) \begin{cases} V_f t & \text{for } 0 < t < (60/n) \\ h & \text{for } \quad t > (60/n) \end{cases} \tag{A.2}$$

where $h$ is the chip width at steady state (after one revolution) given by:

$$h = V_f(60/n) \tag{A.3}$$

In case of tool engagement ($a$) in equation (A.1) is zero.

case ($b$): Change in feedrate (Fig. A.2)

$$A(t) = a[h + \Delta h(t)] \tag{A.4}$$

where $\Delta h(t)$ is given by:

$$\Delta h(t) = \begin{cases} \Delta V_f t & 0 < t < (60/n) \\ \Delta V_f(60/n) & (60/n) < t \end{cases} \tag{A.5}$$

where $\Delta V_f$ if the change in the feedrate.

case ($c$): Change in spindle speed (Fig. A.3)

$$A(t) = a[h + \Delta h(t)] \tag{A.6}$$

where $\Delta h(t)$ is given by:

$$\Delta h(t) = \begin{cases} \Delta V_f(n/n)t & 0 < t < (60/n) \\ \Delta h & (60/n) < t \end{cases} \tag{A.7}$$

where $\Delta n$ is the change in spindle speed during one revolution of the workpiece, $n$ is the average spindle speed during this revolution, and $\Delta h$ is the steady state change in the chip width given by:

$$\Delta h = 60\, V_f(\Delta n/n^2) = h(\Delta n/n) \tag{A.8}$$